Effect of Wood Specimen Length on Uniaxial Stress–Strain Curve along the Grain Direction

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Abstract

The axial compression test was carried out on the basis of three different-size specimens of white wax (*Fraxinus chinensis*), fir (*Abies fabri*), and Yunnan pine (*Pinus yunnanensis*) to explore the influence of specimen length on the compression strength parallel to grain, peak strain, and elastic modulus of wood. Weibull's weakest-link theory, Bazant's law of size effect, and Carpinteri's multiple fractal size effect were used for the analysis. The results showed significant size effects on theses parameters of wood along the grain. Using the three theories to predict the compressive strength of 100-mm-length specimens, the predicted value of Bazant's law of size effect has the smallest error with the measured value, followed by Weibull's weakest-link theory and Carpinteri's law of multiple fractal size effect. The size-effect coefficients of the down-grain compressive strength of the three woods obtained using the slope method were 0.06, 0.11, and 0.09, respectively, which were consistent with the values of *m* in the Weibull fitting function.

Wood as a traditional building material has good mechanical properties and is commonly used for making structural stress members (Zheng et al. 2019, Guo et al. 2022). Clear specimen tests mainly obtain mechanical property parameters of wood, but this method ignores the influence of defects such as knots and cracks, and thus the mechanical parameters obtained are often high. For this reason, researchers have discounted those obtained from clear wood specimens by quantifying wood defects to obtain more realistic mechanical parameters (Jiang et al. 2012). However, some researchers have pointed out that wood has obvious size effects and wood mechanical properties are susceptible to changes in specimen dimensions (Zauner and Niemz 2014, Zhou et al. 2015, Bal 2021, Tapia and Aicher 2022). For this reason, researchers have studied the law of size effect on the mechanical properties of wood, aiming at parameter adjustment by size-effect coefficients to obtain the real mechanical parameters of wood.

Research on size effects on the mechanical properties of wood has focused on five areas: flexural strength, compressive strength, tensile strength, shear strength, and elastic modulus of wood. For example, Madsen and Tomoi (1991) studied the flexural strength of 27 sizes of spruce, pine, and fir and found that specimen length had a large effect and width and thickness had a weak or no effect on flexural strength. Fonselius (1997) studied the size effect of flexural strength of spin-cut laminated wood (LVL) and also found that width had no effect on the flexural strength of the specimens; the combined size-effect coefficient of LVL length and height was 0.075 using the slope method and 0.068 using the shape parameter method, which was in better agreement. Barrett et al. (1995) developed a unified size-effect model on the basis of Weibull's weakest chain theory to evaluate the size-effect coefficients of visually graded wood and used this method to obtain size-effect coefficients of 0.10 and 0.11 for the length and width of Canadian softwoods with smooth grain compressive strength, respectively. Madsen (1990) tested the tensile strength of 38-mm-thick fir wood and found that the tensile strength decreased by 14% when the length of the specimen was increased from 3 to 6 m. When the length of the specimen was increased four times, the strength decreased by 27% compared with the initial strength, and the length and width effect coefficients obtained on the basis of the statistics of the test data were 0.22 and 0.05,

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This paper was received for publication in August 2023. Article no. 23-00046.

Forest Prod. J. 74(1):28–33.
doi:10.13073/FPJ-D-23-00046

Figure 1.—Wood specimens of all sizes.

respectively. Keenan et al. (1985) tested the shear strength of clear wood beams, glued laminated timber beams, and commercial glued laminated timber beams and found that the shear strength of clear wood beams was affected by depth, width, and shear plane; the shear strength of glued laminated timber beams was affected by depth, width and shear-to-span ratio; and in commercial glued laminated timber beams, no significant effect of depth and width on shear strength was found. Jamil et al. (2013) found that the modulus of elasticity of large members was greater than that of small clear specimens. Hu et al. (2019) tested the modulus of elasticity of five sizes of beech specimens and found that it increased with increasing height and decreased with increasing cross-sectional area of the specimens.

When analyzing the size effects of wood, researchers often use Weibull's weakest-link theory (Walley and Rogers 2022) to analyze the size effects of mechanical properties such as compressive and tensile strengths of wood. In addition, some researchers have also analyzed wood size effects using Bazant's size-effect law (Bazant and Pfeiffer 1987) and Carpinteri's multiple fractal size-effect law (Carpinteri and Ferro 1994). However, there are fewer examples of applying these two theories, and few researchers have compared the accuracy of these three theories when describing wood size effects.

The above studies show that the effect of specimen size on the mechanical properties of materials cannot be neglected, but so far, researchers have yet to study the effect law of size effect on the elastic modulus, compressive strength, and peak strain of wood in depth. Given this, this paper uses wax (*Fraxinus chinensis*), fir (*Abies fabri*), and Yunnan pine (*Pinus yunnanensis*) as the original wood to make parallel compression specimens with dimensions of 25 by 25 by 40 mm, 25 by 25 by 60 mm, and 25 by 25 by 80 mm. Axial compression tests on specimens of various sizes and the three theories (Weibull's weakest link, Bazant's law of size effects, and Carpinteri's law of multiple fractal size effects) were used to investigate the effect of specimen length on wood compressive strength, peak strain, and elastic modulus.

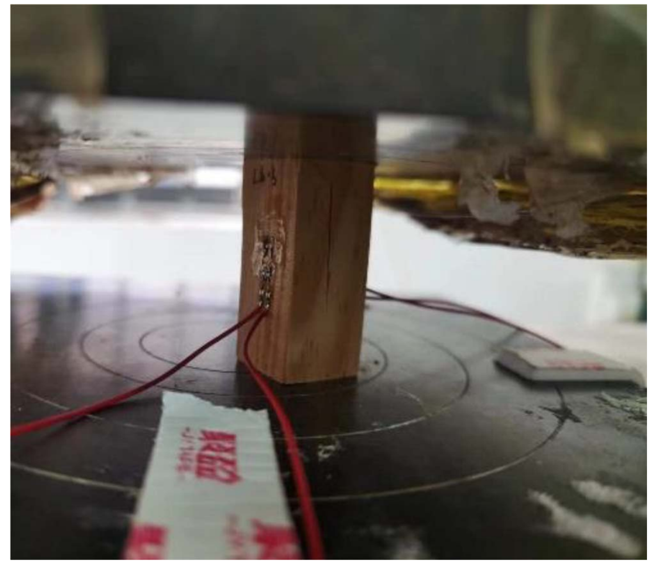


Figure 2.—Testing apparatus.

Materials and Methods

Materials

To analyze size-effect characteristics, three kinds of timber, wax, fir, and Yunnan pine, were selected, fir and Yunnan pine being coniferous and wax being broadleaf. Sawn timber (wax, fir, and Yunnan pine) with dimensions of 40 by 200 by 4,000 mm were selected to carry out the test. The original sawn wood was cut, press planed, and hand planed flat, and then planed into dimensional lumber of 25 by 25 by 700 mm, as shown in Figure 1. The specimens were cut into 25 by 25 by 40-mm, 25 by 25 by 60-mm, and 25 by 25 by 80-mm wood specimens, six specimens of each size. The specimens need to be equilibrated before the test; they were placed in a 25°C room with a relative humidity of 45 percent for 30 days until the specimens reached equilibrium moisture content. The

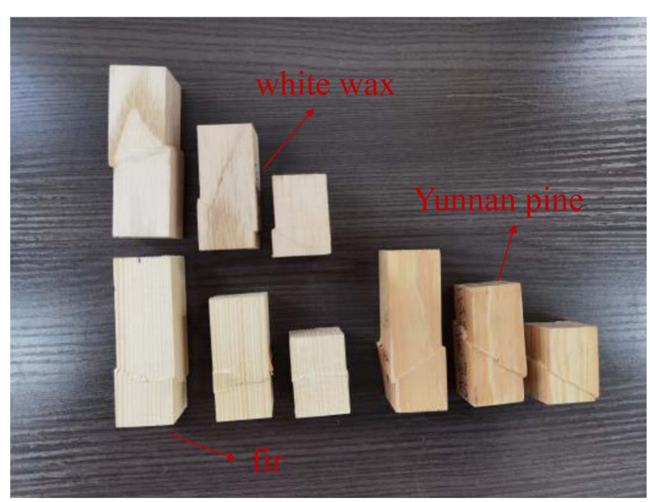
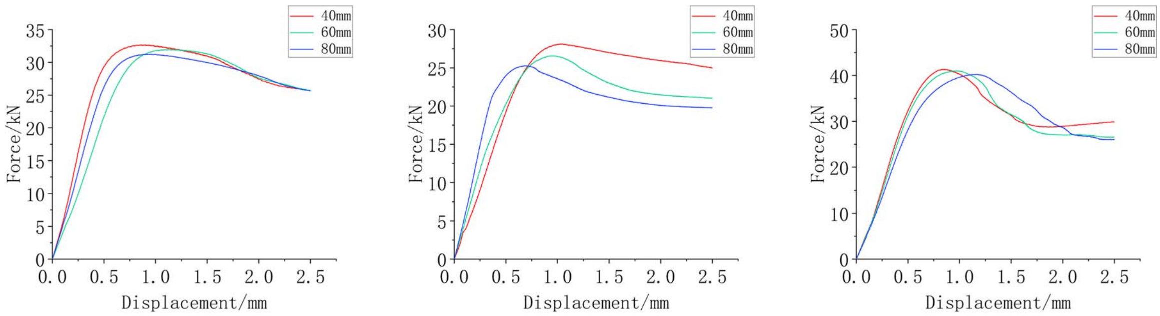


Figure 3.—Specimen failure form.



（a）white wax （b）fir （c）Yunnan pine

Figure 4.—Average load-displacement curves of specimens of different sizes.

test environment was also at a temperature of 25°C and relative humidity of 45 percent. The density of wax specimens was 0.59 g/cm^3 and the moisture content was 12 ± 0.72 percent; the density of fir specimens was 0.41 g/cm^3 and the moisture content was 12 ± 0.97 percent; the density of Yunnan pine specimens was 0.69 g/cm^3 and the moisture content was 12 ± 0.89 percent.

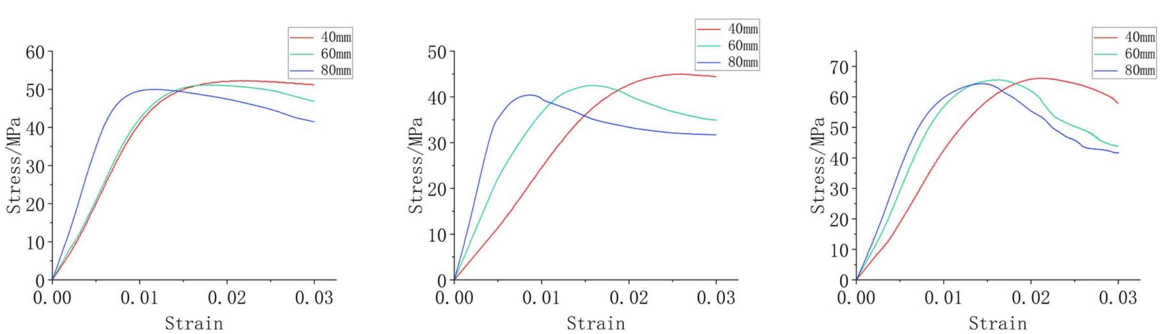
Method

This test refers to the Standard for Test Methods for Wood Structures (GB/T 50329-2012) and the Test Method for Compressive Strength of Wood with Smooth Grain (GB/T 1935-2009), using a 600-kN microcomputer-controlled electrohydraulic servopressure tester to load the wood specimens. The test-loading regime was selected as displacement loading, and to avoid the influence of strain rate on the test results, the loading speed was 1 mm/min. The test setup is shown in Figure 2.

Results

The failure diagrams of three types of wood specimens are shown in Figure 3. The damage mode of the specimens of all sizes of the three types of wood was oblique shear damage. The average load-displacement curves of the three wood specimens is shown in Figure 4. The load-displacement curves of both wax and fir specimens exhibit a rise first, with a constant slope in the elastic range, and after reaching the peak load, the load-displacement curves begin to decline gently. The load-displacement curves of Yunnan pine specimens have an obvious decreasing section after reaching the peak point, and the load-displacement curves tend to level off when the load decreases to about 70 percent of the peak load. The average stress–strain curves for each group of specimens of the three kinds of wood are shown in Figure 5. The overall trends of stress–strain curves of wax and Yunnan pine specimens are relatively similar, and the initial stiffness increases with the increase of specimen length, whereas the stress–strain curves of fir specimens are more influenced by specimen length, and the peak strain magnitude and the trend of stress–strain curves after peak stress are different. However, in general, the longer the length of the specimen, the smaller the area of the stress–strain curve, which may be due to the fact that as the length of the specimen increases, the load required for the folding of the wood fibers decreases, and therefore the compressive strength decreases.

Analysis of variance was performed on the test results and is shown in Tables 1, 2, and 3. All P values are <0.05, which indicates that the specimen's length significantly affects the compressive strength of the specimen with smooth grain. The average values of compressive strength and peak strain for each size specimen of wood were calculated as shown in Table 4, in which the coefficient of variation of compressive strength of all three wood specimens is less than 10 percent, which indicates that the dispersion of test results is small. The mean compressive strength and mean peak strain of wax, fir, and Yunnan pine decreased with increasing specimen length: the mean compressive strength of the three woods decreased by 4, 7, and 6 percent, respectively, and the mean peak strain decreased by 46, 70, and 33 percent, respectively. The average elastic moduli of



（a）white wax （b）fir （c）Yunnan pine

Figure 5.—Average stress–strain curves of specimens of different sizes.

Table 1.—Variance analysis of longitudinal tensile strength of three sizes of white wax specimens.

Source	Sum of squares	df	Mean square	F	P value	F critical value
SSR	13.87	2	6.93	6.05	0.011869	3.68
SSE	17.20	15	1.15			
SST	31.07	17				

SSR was Sum of Squares Regression, SSR was Sum of Squared Errors, SST was Sum of Squares Total.

the three sizes of wax specimens were 3,795, 4,169, and 6,772 MPa, respectively; the average elastic moduli of the three sizes of fir specimens were 2,496, 4,549, and 6,208 MPa, respectively, and the average elastic moduli of the three sizes of Yunnan pine specimens were 4,885, 6,647, and 7,632 MPa, respectively. The average elastic moduli increased by 78, 149, and 56 percent, respectively. The magnitude of influence of specimen size on mechanical parameters of wood is not the same for all wood species.

Length dimension effect

The wood size effect is usually described by Weibull's weakest chain theory, which assumes that the overall strength of a member depends only on the strength of its weakest part, and once the maximum stress reaches the damage limit, the whole member is destroyed, the expression of which is shown in Equation 1:

$$\sigma_N = \sigma_0 \left(\frac{D_0}{D}\right)^m \tag{1}$$

where σ_N is the nominal strength; D is the specimen size; and D_0, m, and σ_0 are constants that can be obtained by fitting the test data.

Fracture mechanics have been used to explain the phenomenon that wood strength decreases with increasing member size; the size-effect theory and approximate nonlinear size-effect model proposed by Bazant are the most representative. The theory suggests that a long crack or a fracture zone containing many microcracks undergoes stable expansion before the material reaches the peak load, and this process is accompanied by the redistribution of stress and the release of fracture energy, the expression of which is shown in Equation 2:

$$\sigma_N = \frac{Bf_t}{\sqrt{1 + \left(\frac{d}{d_0}\right)}} \tag{2}$$

Table 2.—Variance analysis of longitudinal tensile strength of three sizes of fir specimens.

Source	Sum of squares	df	Mean square	F	P value	F critical value
SSR	32.09	2	16.04	9.33	0.002328	3.68
SSE	25.79	15	1.72			
SST	57.88	17				

SSR was Sum of Squares Regression, SSR was Sum of Squared Errors, SST was Sum of Squares Total.

Table 3.—Variance analysis of longitudinal tensile strength of three sizes of Yunnan pine specimens.

Source	Sum of squares	df	Mean square	F	P value	F critical value
SSR	47.42	2	23.71	23.4	2.42×10^{-5}	3.68
SSE	15.17	15	1.01			
SST	62.60	17				

SSR was Sum of Squares Regression, SSR was Sum of Squared Errors, SST was Sum of Squares Total.

where σ_N is the nominal strength. Bf_t and d_0 are constants that can be obtained by fitting the test data, and d is the specimen size. The empirical constant d_0 contains the length of the fracture zone; d/d_0 denotes the brittleness of the specimen, and when $d/d_0 \rightarrow 0$, the material is plastic; when $d/d_0 \rightarrow \infty$, the material is completely brittle.

For brittle materials, the evolution of internal microcracks is characterized by fractals, on which Carpinteri proposed the theory of multiple fractal size effects. The theory suggests that the differences in the fractal characteristics of microcracks under different observation levels are the reason for the size effect. Its expression is shown in Equation 3:

$$\sigma_N = \sqrt{A + \frac{B}{D}} \tag{3}$$

where σ_N is the nominal strength. A and B are constants that can be obtained by fitting the test data, and D is the specimen size.

The compressive strength data for each size specimen of the three woods in Table 4 were fitted using the three theories, and the fitting results are shown in Figure 6. The fitted curves of all three size-effect theories are good. The values of each parameter in the three theories can be found by fitting, and the results are shown in Table 5. For further validation, wood specimens with dimensions of 25 by 25 by 100 mm were fabricated for axial compression tests on the basis of each of the three types of wood. The compressive strengths of the three wood specimens were 50.05, 40.32, and 62.53 MPa, respectively. The compressive strength of 25 by 25 by 100-mm wood specimens was predicted on the basis of three theories: Weibull's weakest-link theory, Bazant's law of size effect, and Carpinteri's law of multiple fractal size effect, and the results are shown in Table 6. The predicted strength values of all three theories were close to the measured values, indicating that all three theories can be used to describe the size effects of wood. Regarding the error between the predicted and measured values, Bazant's size-effect law has the least error, Weibull's weakest-link theory follows, and Carpinteri's multiple fractal size-effect law has the largest error. Those data indicate that Bazant's size-effect law is more suitable for describing the size effect of wood compressive strength.

Barrett (1974), on the basis of Weibull's weakest-link theory, argued that the size effect on the compressive strength of wood can be expressed as:

$$\frac{f_0}{f} = \left(\frac{V}{V_0}\right)^s \tag{4}$$

In Equation 4, f, and f_0 are the compressive strengths of wood specimens at volume V and V_0, respectively, and s is the size-effect coefficient.

Table 4.—Test results of Tables 1–3.

Wood species	Group	Size, mm	Average compressive strength, MPa	Intensity coefficient of variation, %	Average peak strain	Average modulus of elasticity, MPa
White wax	A	$25 \times 25 \times 40$	52.48	2.16	0.021	3,795
	B	$25 \times 25 \times 60$	51.35	3.40	0.018	4,169
	C	$25 \times 25 \times 80$	50.33	1.11	0.011	6,772
Fir	D	$25 \times 25 \times 40$	45.45	4.18	0.029	2,496
	E	$25 \times 25 \times 60$	43.74	3.25	0.015	4,549
	F	$25 \times 25 \times 80$	42.18	4.36	0.009	6,208
Yunnan pine	G	$25 \times 25 \times 40$	68.63	1.72	0.022	4,885
	H	$25 \times 25 \times 60$	66.52	2.10	0.017	6,647
	I	$25 \times 25 \times 80$	64.66	0.71	0.014	7,632

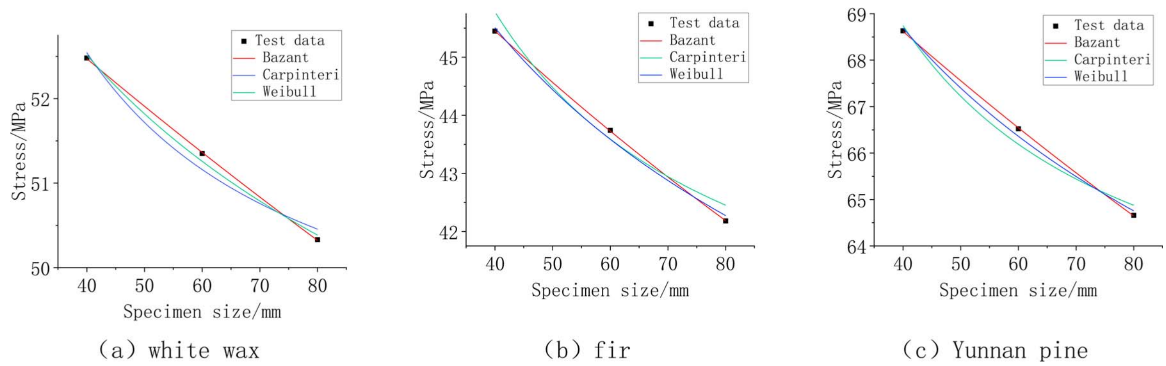


Figure 6.—Measured results of compressive strength of three types of wood and fitted curves of three size effect equations.

Since the wood specimen is rectangular in this test, Equation 4 can be written as

$$\frac{f_0}{f} = \left(\frac{V}{V_0}\right)^{s} = \left(\frac{h}{h_0}\right)^{s_h} \left(\frac{b}{b_0}\right)^{s_b} \left(\frac{L}{L_0}\right)^{s_l} \tag{5}$$

In Equation 5, b, b_0 is the width of the wood specimen under volume V, V_0. L, L_0 is the length of the wood specimen under volume V, V_0. h, h_0 is the thickness of the wood specimen under volume V, V_0. s_b, s_l, and s_h are the coefficients of the size effects of the width, length, and thickness of the wood specimen.

Since the cross-sectional areas of the three sizes of wood specimens are the same, Equation 5 can be simplified as follows:

$$\frac{f_0}{f} = \left(\frac{V}{V_0}\right)^{s} = \left(\frac{L}{L_0}\right)^{s_l} \tag{6}$$

Taking the logarithm of Equation 6, the length-dimension-effect coefficient is obtained as

$$s_l = \frac{(\lg f_0 - \lg f)}{(\lg L - \lg L_0)} \tag{7}$$

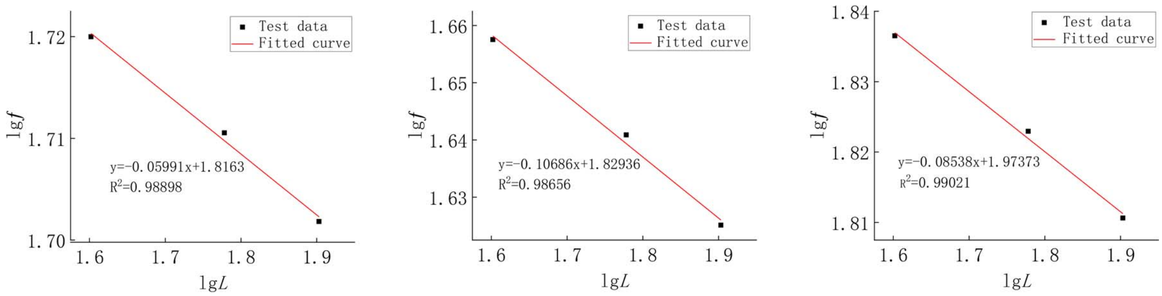
This method is also known as the slope method. Equation 7 was used to analyze the linear regression of the compressive strength of the three sizes of ash, fir, and Yunnan pine wood specimens in Table 4, respectively, and the results of the calculations are shown in Figure 7. The logarithms of the compressive strength of the specimens of ash, fir, and Yunnan pine wood all decrease with the increase of the logarithm of the length. The length size-effect coefficients of compressive strength of the three kinds of wood are 0.06, 0.11, and 0.09 (after taking two decimals), respectively, which agree with the values of m in Table 6. The data indicate that the length size-effect coefficient of the longitudinal compressive strength of wood can be determined by fitting the parameter m in Weibull's weakest chain theory.

Table 5.—Parameter values for the three theories (Weibull's weakest-link theory, Bazant's law of size effect, and Carpinteri's law of multiple fractal size effect).

Theory	Parameters	White wax	Fir	Yunnan pine
Weibull	σ_0	40	43	45
	d_0	3,793	68	5,736
	M	0.06	0.11	0.09
Bazant	Bf_t	55	50	73
	d_0	418	208	276
Carpinteri	A	2,330	1515	3,692
	B	17,224	22,393	41,355

Table 6.—Test and predicted values of the three theories (Weibull's weakest-link theory, Bazant's law of size effect, and Carpinteri's law of multiple fractal size effect).

	Test value, MPa	Weibull fit, MPa	Error	Bazant fit, MPa	Error	Carpinteri fit, MPa	Error
White wax	50.05	49.72	−0.67%	49.34	−0.76%	50.03	−0.05%
Fir	40.32	41.28	2.38%	40.80	1.91%	41.70	3.42%
Yunnan pine	62.53	63.54	1.61%	62.90	1.06%	64.07	2.47%



(a) white wax (a) fir (a) Yunnan pine

Figure 7.—Relationship between compressive strength and specimen length.

Conclusions

In this paper, the axial compression test was carried out on wax, fir, and Yunnan pine wood to study the effect of specimen length on compressive strength, peak strain, and elastic modulus. The size effect of wood compressive strength was analyzed on the basis of three theories, Weibull's weakest-link theory, Bazant's law of size effect, and Carpinteri's law of multiple fractal size effect, and the following three conclusions were obtained:

1. Specimen length significantly affects wood smooth-grain compressive strength, peak strain, and elastic modulus. When the specimen length increased from 40 to 80 mm, the decreases in average compressive strength of the three wood specimens were 4.10, 7.19, and 5.78 percent, respectively; the decreases in average peak strain were 46.38, 70.07, and 33.49 percent, respectively; the average elastic modulus increases were 78.45, 148.72, and 56.23 percent, respectively.
2. Weibull's weakest-link theory, Bazant's law of size effect, and Carpinteri's law of multiple fractal size effect all describe the size effect of wood compressive strength better when predicting the compressive strength of 100-mm-length specimens; the predicted value of Bazant's law of size effect has the smallest error with the measured value, followed by Weibull's weakest chain theory and Carpinteri's law of multiple fractal size effect, with the largest error.
3. The size-effect coefficients of the three woods for sagittal compressive strength obtained using the slope method are 0.06, 0.11, and 0.09, which are consistent with the values of m in the Weibull fitting function; therefore, the size-effect coefficients of the sagittal compressive strength for the length of the woods can be determined by fitting the parameter m in Weibull's weakest link theory.

Acknowledgments

The authors gratefully acknowledge the funding support received from the National Natural Science Foundation of China (grant no. 52268051).

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