# Examination of the Specimen Configuration and Analysis Method in the Flexural and Longitudinal Vibration Tests of Solid Wood and Wood-Based Materials

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## Abstract

The Young's modulus and shear modulus of solid wood (Sitka spruce, *Picea sitchensis*), medium-density fiberboard (MDF), and Lauan wood (*Shorea* sp.) with five-ply construction were determined by conducting flexural and longitudinal vibration tests with various specimen depth/length ratios and performing a subsequent finite element analysis (FEA). The values of Young's modulus and shear modulus were calculated by three analysis methods: (1) the method based on the rigorous solution of Timoshenko's differential equation (*Phil. Mag*. 41:744–746, 1921), (2) the iteration procedure proposed by Hearmon (*Brit. J. Appl. Phys*. 9:381–388, 1958), and (3) the method in which Young's modulus measured by the longitudinal vibration test is substituted into an approximated equation proposed by Goens (*Ann. Physik. Ser. 7* 11:649–678, 1931). The results obtained from the FEA suggested that the analysis method does not influence the values of Young's modulus or shear modulus. However, the results obtained indicated that the analysis method influenced the measured values of these moduli. Although Method 3 is simpler than Methods 1 and 2, the influence of depth/span ratio was more pronounced when using resonance frequencies lower than the second flexural vibration mode. When using the resonance frequency for flexural vibrations higher than the third mode, however, it is promising that the shear modulus can be measured while reducing the influence of the depth/length ratio.


Solid wood and wood-based materials are often used in construction projects. Obtaining reliable strength and deformation properties for these materials, including their Young's modulus and shear modulus, is therefore essential to ensuring that construction procedures are efficient and cost-effective.

Among the experimental methods used to determine the Young's modulus and shear modulus of solid wood and wood-based materials, such as medium-density fiberboard (MDF) and plywood, the vibration method is very effective, and there are many examples of measuring the Young's modulus and shear modulus of these materials by this method. The longitudinal vibration method is effective in measuring the Young's modulus because of its simplicity; thus, there are many examples of measuring the Young's modulus of solid wood and wood-based materials by the longitudinal vibration method (Ono and Norimoto 1985, Sobue 1986b, Haines et al. 1996, Ilic 2003, Yang et al. 2003, Yoshihara 2012c). In contrast, the torsional vibration method is valid in measuring the shear modulus because the pure shear stress condition can be induced with this method. Therefore, several studies have used the torsional vibration method to measure the shear modulus (Becker and Noack 1968, Nakao et al. 1985, Nakao and Okano 1987, Sobue 1999, Tonosaki et al. 2010). Nevertheless, it is often difficult to measure the shear modulus of orthotropic materials such as wood and wood-based materials with the torsional vibration method because the vibration is generated three-dimensionally such that the resonance frequency for the torsional vibration mode inevitably contains the effect of two shear moduli in the side planes surrounding the torsional axis. To measure the shear modulus of a single plane by the torsional vibration method, it is necessary to conduct multiple vibration tests using specimens with various aspect ratios along their

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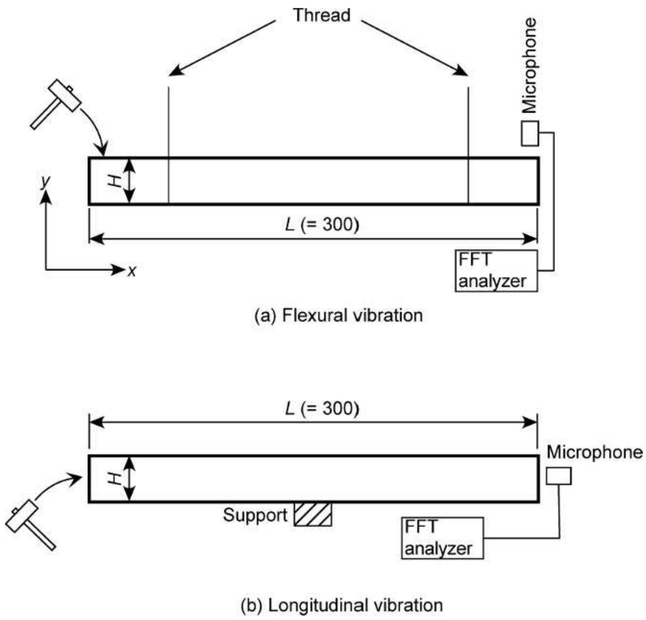


*Figure 1.—Diagram of the flexural (a) and longitudinal (b) vibration tests.*

cross section to separate the shear moduli from each other (Nakao et al. 1985). Otherwise, a plate-shaped specimen is used to determine the influence of the plane along which the shear modulus is measured (Nakao and Okano 1987, Yoshihara 2009) or the differences between the shear moduli are ignored (Tonosaki et al. 2010).

When comparing the longitudinal and torsional vibration methods, the flexural vibration method is advantageous because the Young's modulus and shear modulus can be determined simultaneously and are easily obtained. In addition, the flexural vibration method is free from the geometrical problem described above that is associated with the torsional vibration method of measuring the shear modulus, as well as the Young's modulus, of solid wood by flexural vibration methods based on Timoshenko's vibration theory (Hearmon 1958, 1966; Nakao 1984; Sobue 1986a; Chui and Smith 1990; Kubojima et al. 1996, 1997; Divós et al. 1998, 2005; Brancheriau and Baillères 2002, 2003; Brancheriau 2006; Murata and Kanazawa 2007; Tonosaki et al. 2010; Khademi-Eslam et al. 2011; Sohi et al. 2011). When measuring the in-plane shear modulus of MDF and plywood, however, the influence of specimen configuration is so significant that the in-plane shear modulus cannot be obtained accurately from flexural vibration tests if the depth of the specimen is not large enough relative to its length (Yoshihara 2011, 2012a); therefore, the specimen configuration should be appropriately determined to measure the in-plane shear modulus accurately by flexural vibration tests.

In addition to the configuration of the specimen used for vibration tests, the method used to analyze the data obtained from vibration tests influences the accuracy of the Young's modulus and shear modulus measured from solid wood and wood-based materials. As in the examples described above (Hearmon 1958, 1966; Nakao 1984; Sobue 1986a; Chui and Smith 1990; Kubojima et al. 1996, 1997; Brancheriau and Baillères 2002, 2003; Divós et al. 1998, 2005; Brancheriau 2006; Murata and Kanazawa 2007; Tonosaki et al. 2010; Khademi-Eslam et al. 2011; Sohi et al. 2011), the Young's modulus and shear modulus were determined based on the iteration method proposed by Hearmon (1946), the details of which are described below. In previous studies (Yoshihara 2011, 2012a), the Young's modulus and shear modulus of MDF and plywood were numerically obtained from the rigorous solution of Timoshenko's equation derived by Goens (1931), the details of which are also described below. As shown in these examples, there are several methods used to analyze the data obtained from vibration tests. Nevertheless, a definitive method has not been determined yet, although there is concern that incorrectly adopted analysis methods induce inaccurate measurements of the Young's modulus and shear modulus of wood and wood-based materials.

As described above, it is essential to determine the Young's modulus and shear modulus values precisely in order to ensure that construction procedures where the solid wood and wood-based materials are used are efficient and cost-effective, so appropriate methods must be established for this purpose. Additionally, the method that is adopted should be simple to conduct when considering the practical measurement of these moduli. To establish the definitive method for measuring the Young's modulus and shear modulus of solid wood and wood-based materials by a vibration method, the comparative study on the analysis methods, which had not been conducted in previous studies at all, is very important.

In this work, free-free flexural vibration tests were performed on solid wood, MDF, and plywood specimens with various depth/length ratios, and the Young's modulus and the shear modulus were obtained from three different analysis methods, which are based on Timoshenko's vibration theory.

## Flexural Vibration Equation Based on Timoshenko's Theory

Figure 1a shows a diagram of the flexural vibration test. In 1921, Timoshenko presented the following differential equation of flexure, in which the shear deflection and rotary inertia were taken into account:

$$E_x I \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} - \rho I \left(1 + \frac{sE_x}{G_{xy}}\right) \frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{s\rho^2 I \partial^4 y}{\partial t^4} = 0 \quad (1)$$

where

$E_x$ = Young's modulus in the $x$ direction,

$G_{xy}$ = shear modulus in the $xy$ plane,

$I$ = secondary moment of inertia,

$A$ = cross-sectional area,

$\rho$ = density of the beam, and

$s$ = Timoshenko's shear factor.

When the specimen has a rectangular cross section with width and height $B$ and $H$, respectively, $A$ and $I$ are derived as $BH$ and $BH^3/12$, respectively. Additionally, the value of $s$ is approximately 6/5 for a beam with a rectangular cross section when the geometrical axis coincides with the orthotropic axis. When the geometrical and orthotropic axes do not coincide with each other, the $s$ value depends on the angle between these axes (Yoshihara 2012b).

The rigorous solution to Equation 1 under the free-free flexural condition was derived by Goens (1931) as follows:

$$\begin{cases} \dfrac{\cot\dfrac{k_n}{2}\sqrt{\sqrt{\beta^2k_n^4+1}+\alpha k_n^2}}{\coth\dfrac{k_n}{2}\sqrt{\sqrt{\beta^2k_n^4+1}-\alpha k_n^2}} \\ \quad = -\dfrac{\sqrt{\sqrt{\beta^2k_n^4+1}+\alpha k_n^2}}{\sqrt{\sqrt{\beta^2k_n^4+1}-\alpha k_n^2}}\cdot\dfrac{\sqrt{\beta^2k_n^4+1}-\beta k_n^2}{\sqrt{\beta^2k_n^4+1}+\beta k_n^2} \\ \text{(symmetric mode)} \\ \dfrac{\tan\dfrac{k_n}{2}\sqrt{\sqrt{\beta^2k_n^4+1}+\alpha k_n^2}}{\tanh\dfrac{k_n}{2}\sqrt{\sqrt{\beta^2k_n^4+1}-\alpha k_n^2}} \\ \quad = \dfrac{\sqrt{\sqrt{\beta^2k_n^4+1}+\alpha k_n^2}}{\sqrt{\sqrt{\beta^2k_n^4+1}-\alpha k_n^2}}\cdot\dfrac{\sqrt{\beta^2k_n^4+1}-\beta k_n^2}{\sqrt{\beta^2k_n^4+1}+\beta k_n^2} \\ \text{(antisymmetric mode)} \end{cases} \tag{2}$$

where $n$ is the mode number and $\alpha$ and $\beta$ are derived as follows:

$$\begin{cases} \alpha = \dfrac{6}{L^2H^2}\left(\dfrac{sE_x}{G_{xy}}+1\right) \\ \beta = \dfrac{6}{L^2H^2}\left(\dfrac{sE_x}{G_{xy}}-1\right) \end{cases} \tag{3}$$

When the resonance frequency for the $n$th flexural vibration mode is defined as $f_n$, $k_n$ is derived as follows:

$$k_n = \sqrt[4]{\frac{48\pi^2\rho f_n^2}{E_xH^2}}L \tag{4}$$

The values of $E_x$ and $G_{xy}$ corresponding to each resonance mode can be simultaneously obtained from the numerical solution to Equation 2. To solve Equation 2, however, a specialized mathematical software package such as Mathematica (Yoshihara 2011, 2012a, 2012b) is required.

Approximation methods are an effective way to reduce the inconvenience of using the mathematical software package. Goens (1931) also derived an approximation equation from Equation 2 as follows:

$$\begin{aligned} \frac{m_n^4}{k_n^4} &= 1+\frac{H^2}{12L^2}\left[6m_nF(m_n)+m_n^2F^2(m_n)\right] \\ &\quad +\frac{sE_xH^2}{12G_{xy}L^2}\left[-2m_nF(m_n)+m_n^2F^2(m_n)\right]-\frac{\pi^2 s\rho H^2 f_n^2}{3G_{xy}} \end{aligned} \tag{5}$$

where the coefficients $m_n$ and $F(m_n)$, which correspond to each resonance mode, are given by

$$\begin{cases} m_1 = 4.730 \\ m_2 = 7.853 \\ m_n = \dfrac{(2n+1)\pi}{2}(n \geq 3) \end{cases} \tag{6}$$

and

$$\begin{cases} F(m_1) = 0.9285 \\ F(m_2) = 1.0008 \\ F(m_n) = 1(n \geq 3) \end{cases} \tag{7}$$

When the multiple resonance frequencies for the flexural modes are measured, the values of $E_x$ and $G_{xy}$ can be determined by the iteration method proposed by Hearmon (1958). In this method, $X$ and $Y$ corresponding to each mode are calculated using Equation 5 as follows:

$$\begin{cases} X = \dfrac{4\pi^2\rho L^2 f_n^2}{m_n^4}\left[-2m_nF(m_n)+m_n^2F^2(m_n)\right] \\ Y = \dfrac{4\pi^2\rho L^2 f_n^2}{m_n^4} \\ \qquad \cdot\left[\dfrac{12L^2}{H^2}+6m_nF(m_n)+m_n^2F^2(m_n)-\dfrac{4\pi^2 sL^2\rho f_n^2}{G_{xy}}\right] \end{cases} \tag{8}$$

The $X$–$Y$ relation corresponding to each mode is regressed into the linear function $Y = q - pX$, and the $E_x$ and $G_{xy}$ values are determined by the value of $q$ and $sq/p$, respectively. There are many examples of obtaining the $E_x$ and $G_{xy}$ values of solid wood according to this method (Hearmon 1958, 1966; Nakao 1984; Sobue 1986a; Chui and Smith 1990; Brancheriau and Baillères 2002; Brancheriau 2006; Murata and Kanazawa 2007; Yoshihara 2009; Tonosaki et al. 2010; Khademi-Eslam et al. 2011; Sohi et al. 2011). Nevertheless, the $E_x$ and $G_{xy}$ values are not explicitly contained in Equation 5, and they should be calculated by the iteration procedure, the details of which are described below.

Here, another simple method using Equation 5 is proposed using the Young's modulus value obtained from the longitudinal vibration test. As denoted in previous articles (Neuhaus 1983, Garab et al. 2010), wood and wood-based materials are not ideally orthortropic materials, so the Young's moduli obtained from the flexural and longitudinal vibration methods often deviate from each other. When the Young's modulus value obtained from the longitudinal vibration method can be used instead of that obtained from the flexural vibration method, however, it is feasible to obtain the shear modulus easily by the following procedure. Initially, a longitudinal vibration test is conducted, and the $E_x$ value is obtained by substituting the fundamental frequency of longitudinal vibration mode $f_L$ into the following equation:

$$E_x = 4f_L^2 L^2 \rho \tag{9}$$

Then the $E_x$ value is substituted into the following equation, which is transformed from Equation 5:

$$G_{xy} = \frac{\frac{H^2}{12L^2}\left[sF_2(m_n)E_x - 4\pi^2 s\rho L^2 f_n^2\right]}{\frac{m_n^4}{k_n^4} - \left[1 + \frac{H^2}{12L^2}F_1(m_n)\right]} \tag{10}$$

where

$$\begin{cases} F_1(m_n) = 6m_nF(m_n) + m_n^2F^2(m_n) \\ F_2(m_n) = -2m_nF(m_n) + m_n^2F^2(m_n) \end{cases} \tag{11}$$

Mead and Joannides (1991) and Kubojima et al. (1996, 1997) calculated the $G_{xy}$ value corresponding to each flexural vibration mode by substituting the $E_x$ value obtained from a longitudinal vibration test into Equation 2. As described above, however, the use of Equation 2 is inconvenient because a mathematical software package is required. By substituting the $E_x$ value obtained from the longitudinal vibration test into Equation 10, however, the $G_{xy}$ value can be determined without using any mathematical software package to solve the differential equation or any program for the iteration procedure.

## Finite Element Analysis

In previous studies on the vibration properties of MDF and plywood, the finite element analyses (FEAs) revealed the dependence of the in-plane shear modulus on the depth/length ratio of the specimen (Yoshihara 2011, 2012a). Therefore, it was expected that the influence of the analysis method on the measurement of Young's modulus and shear modulus values could be revealed by FEA.

Three-dimensional (3D) FEA was performed independently of the flexural vibration test using the FEA program ANSYS 12.0. Figure 2 shows the finite element mesh of the specimen. In the analyses, solid spruce, MDF, and five-ply Lauan wood (*Shorea* sp.) models were simulated. The model had a length, *L,* of 300 mm. The depth, *H,* was varied from 10 to 60 mm in 10-mm increments. The width, *T,* was given a value of 9 mm. The model consisted of eight-node brick elements. The finite element mesh was homogeneously divided in the solid spruce and MDF models, and an element had the dimensions of 5, 0.75, and *H*/20 mm in the length, width, and depth directions, respectively. In contrast, the element in the five-ply Lauan wood model had the dimensions of 5 mm and *H*/20 mm in the length and depth directions, respectively. In the width direction, however, the length of the element in the surface and core veneers was 0.6 mm, whereas that in the veneer adjacent to the surface veneer was 0.675 mm.

Table 1 shows the assumed elastic constants for solid Sitka spruce (*Picea sitchensis* Carr.), MDF, and Lauan veneer. The values of the elastic constants of spruce and MDF were obtained from several previous studies by conducting the flexural vibration tests (Yoshihara 2011, Yoshihara and Nakano 2011). In contrast, it was difficult to measure the elastic constants of the Lauan veneer of which the actual plywood panel was composed. Therefore, the elastic constants of the Lauan veneer in Table 1 were taken from the data published in the *Wood Industry Handbook* (Forestry and Forest Products Research Institute 2004). The densities of the spruce, MDF, and Lauan plywood models were 380, 650, and 500 kg/m$^3$, respectively. In the analyses of the Lauan plywood, the following two models were simulated in terms of wood

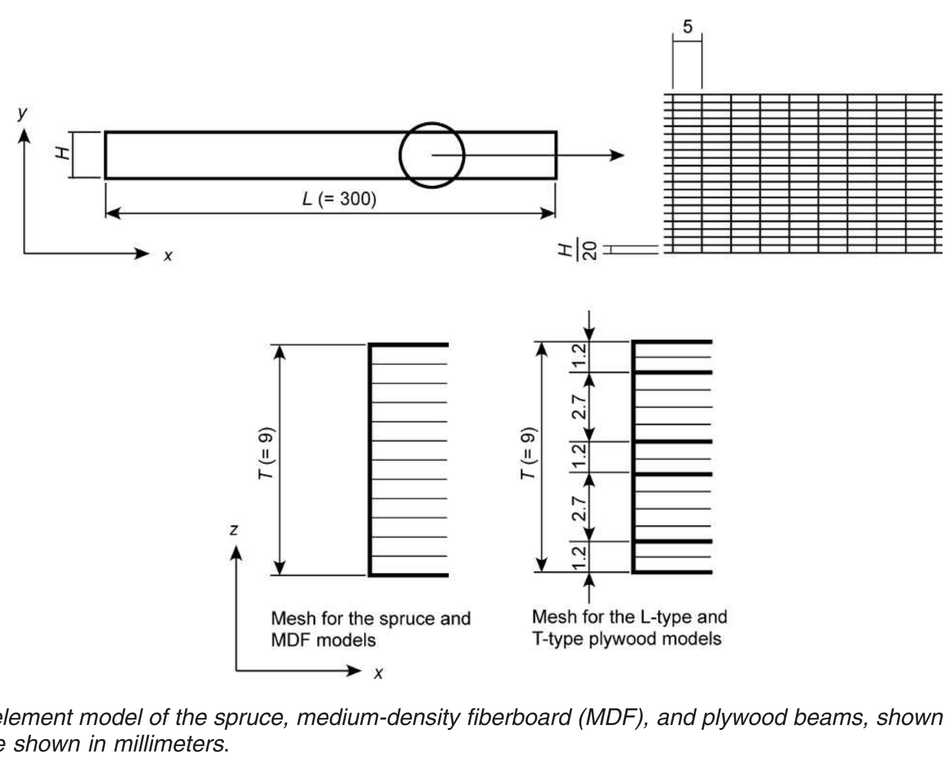


*Figure 2.—Finite element model of the spruce, medium-density fiberboard (MDF), and plywood beams, shown along the* xy *and* xz *planes. Values are shown in millimeters.*

Table 1.—Elastic properties used for the finite element analysis calculations.[a]

| | Young's modulus (GPa) | | | Shear modulus (GPa) | | | Poisson's ratio | | |
|---|---|---|---|---|---|---|---|---|---|
| | $E_x$ | $E_y$ | $E_z$ | $G_{xy}$ | $G_{yz}$ | $G_{xz}$ | $\nu_{xy}$ | $\nu_{yz}$ | $\nu_{xz}$ |
| Solid spruce | 10.8 | 0.42 | 0.64 | 0.65 | 0.03 | 0.58 | 0.49 | 0.32 | 0.39 |
| MDF | 3.0 | 3.0 | 0.30 | 1.18 | 0.30 | 0.30 | 0.28 | 0.30 | 0.30 |
| Lauan veneer | 12.9 | 0.51 | 1.00 | 0.48 | 0.11 | 0.65 | 0.61 | 0.31 | 0.39 |

[a] $x$, $y$, and $z$ directions of solid spruce and Lauan veneer models represent the longitudinal, tangential, and radial directions, respectively, whereas those of medium-density fiberboard (MDF) model represent the length, width, and thickness directions of the sheet, respectively.

grain: (1) the length direction of the surface veneer equals the longitudinal direction, and (2) the length direction of the surface veneer equals the tangential direction. They were defined as the L- and T-type models, respectively. According to classical lamination theory, the Young's modulus along the length direction is derived as follows:

$$\begin{cases} E_L = \dfrac{E_x T_a + E_y T_b}{T_a + T_b} \\ E_T = \dfrac{E_y T_a + E_x T_b}{T_a + T_b} \end{cases} \tag{12}$$

where

$E_L$ = Young's modulus of L-type model in the length direction,

$E_T$ = Young's modulus of T-type model in the length direction,

$T_a$ = total thickness of core and surface veneers, and

$T_b$ = total thickness of the veneers adjacent to the core veneer.

Therefore, $E_L$ and $E_T$ should be 5.47 and 7.94 GPa. In contrast, the value of the in-plane shear modulus should be 0.48 GPa for both models.

Modal analyses were conducted, and the resonance frequencies of the first to fourth flexural vibration modes and the resonance frequency of the first longitudinal vibration mode were extracted, the values of Young's modulus and shear modulus, $E_x$ and $G_{xy}$, respectively, were determined from the following three procedures. (1) The $E_x$ and $G_{xy}$ terms were calculated from the solution to Equation 5 using Mathematica 6. The $G_{xy}$ values corresponding to each vibration mode were calculated by altering the value of $E_x$, and the coefficient of variation (COV) of the $G_{xy}$ values was determined. The $E_x$ value that generates the minimum COV of the $G_{xy}$ values and the mean value of $G_{xy}$ can be regarded as being the most feasible, as described above (Yoshihara 2011, 2012a, 2012b). (2) The $E_x$ and $G_{xy}$ terms were calculated from the iteration using Equation 8 and the resonance of the first to fourth modes. Initially, a virtual value of $G_{xy}$ was substituted into $Y$ of Equation 8, and the refined value of $G_{xy}$ was obtained since $sq/p$ was again substituted into $Y$ (Hearmon 1958). The iteration procedure was conducted using the function incorporated into Microsoft Excel version 14.1.4. The procedure was halted after all values in the formulas changed by less than 0.001 between the iterations. (3) The $E_x$ value was calculated by substituting the fundamental frequency of longitudinal vibration mode $f_L$ into Equation 9. Then the $G_{xy}$ values corresponding to each vibration mode were calculated by substituting the $E_x$ obtained from the longitudinal vibration test and the resonance frequency corresponding to each flexural vibration mode $f_n$ into Equation 10, and the mean value of $G_{xy}$ was obtained.

The $E_x$ values predicted by the three different procedures described above were compared with those input in the FEA program, which were 10.8, 3.0, 5.47, and 7.94 GPa for the solid spruce, MDF, L-type plywood, and T-type plywood models, respectively. Similarly, the $G_{xy}$ values obtained by the three procedures were compared with the input moduli: 0.65, 1.18, and 0.48 GPa for the solid spruce, MDF, and both plywood models, respectively.

## Experimental Validation

### Materials

Sitka spruce (*Picea sitchensis* Carr.) lumber, MDF, and Lauan wood with five-ply construction were used as the test specimens in this study. The densities (±standard deviations) at 12 percent moisture content were 375 ± 5, 657 ± 5, and 496 ± 3 kg/m$^3$, respectively. The samples of Sitka spruce contained 3 to 4 annual rings per 10 mm along the radial direction; the rings were flat enough that their curvature could be ignored. This lumber was free of defects such as knots or grain distortions, and therefore the specimens cut from it could be regarded as ''small and clear.'' The MDF and plywood sheets were fabricated by Ueno Mokuzai Kogyo Co. (Himeji, Japan) and had initial lengths, widths, and thicknesses of 1,820, 910, and 9 mm, respectively. The MDF was fabricated of softwood (typical fiber length of 2 to 4 mm) and urea-formaldehyde resin. The thickness of the surface and core veneers was 1.2 mm, whereas the thickness of the veneer adjacent to the surface veneer was 2.7 mm. The spruce lumber samples were cut to 1,000, 140, and 100 mm in the longitudinal, tangential, and radial directions, respectively, whereas the MDF and plywood sheet samples were cut to 450, 450, and 9 mm in the length, width, and thickness directions, respectively. The samples were stored in a room at a constant temperature of 20°C and 65 percent relative humidity before the test to reach an equilibrium moisture content of approximately 12 percent.

These samples were cut into specimens with the initial length, depth, and width dimensions of 300, 60, and 9 mm, respectively. For the spruce specimens, these directions coincided with those along the longitudinal, tangential, and radial directions, respectively. For the MDF specimens, the length direction was made to coincide with the length direction of the sheet. A plywood specimen with a length direction coincident with the longitudinal direction of the surface veneer was defined as an L-type specimen, whereas that with a length direction coincident with the tangential direction of the surface veneer was defined as a T-type

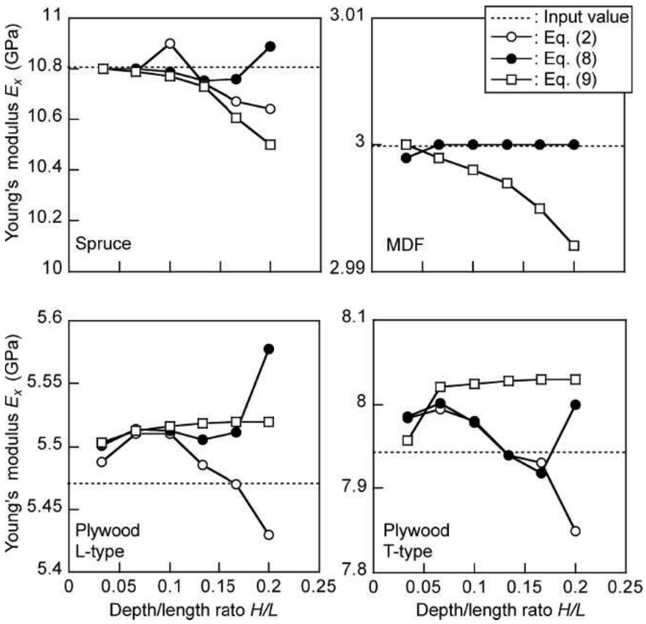


*Figure 3.—Influence of the analysis method on the relationship between the Young's modulus* $E_x$ *obtained by finite element analysis and depth/length ratio* H/L. *Input values of* $E_x$ *correspond to 5.47 and 7.94 GPa for the L- and T-type plywood models, respectively, which were obtained based on classical lamination theory. MDF = medium-density fiberboard.*

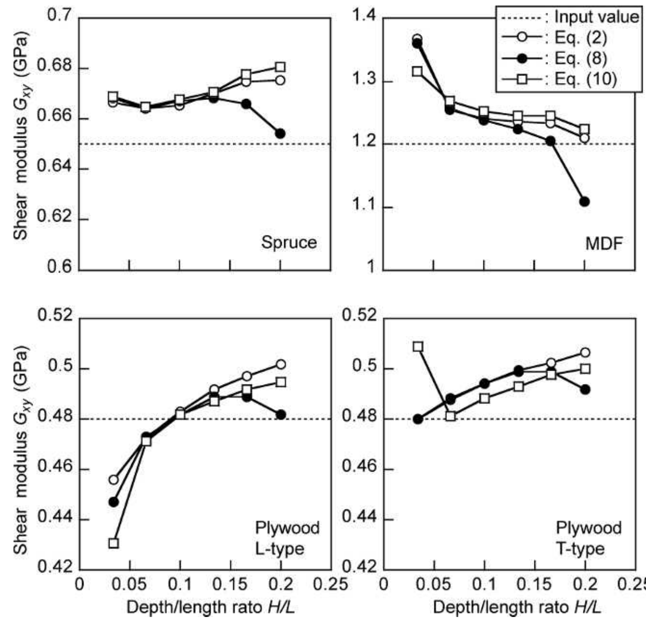


*Figure 4.—Influence of the analysis method on the relationship between the shear modulus* $G_{xy}$ *obtained by finite element analysis and depth/length ratio* H/L. *The input value of* $G_{xy}$ *corresponds to 0.48 GPa for both the L- and T-type plywood models, which coincides with that of the Lauan model shown in Table 1. MDF = medium-density fiberboard.*

specimen. These definitions are similar to those for the FEA. Ten specimens were tested under each test condition.

After conducting the flexural and longitudinal vibration tests described below, the depth of the specimen was reduced, and the succeeding series of vibration tests was conducted using the specimens with reduced depth. The depth ($H$) of the specimens was reduced from 60 to 10 mm at intervals of 10 mm.

## Flexural and longitudinal vibration tests

In the flexural vibration tests, the specimen was suspended by threads at the nodal positions of the free-free resonance vibration mode $f_n$ and excited along the depth direction with a hammer (Fig. 1a). The suspended points were the outermost positions of each vibration mode. In the flexural vibration test, it was difficult to measure the resonance frequencies above the fifth mode because of the small amplitude and, therefore, the first- to fourth-mode resonance frequencies were measured. In the longitudinal vibration tests, the specimen was supported by soft foam at the midlength and excited along the length direction with a hammer (Fig. 1b). The fundamental frequency of the longitudinal vibration mode, defined as $f_L$, was measured. The resonance frequencies were analyzed using a Fast Fourier Transform analysis program.

As in the FEA, the $E_x$ and $G_{xy}$ values were calculated using the three procedures, the details of which are described above. When analyzing the MDF specimens with depths of 10 and 20 mm according to Equation 5, however, the data obtained from the first and second modes and the first mode only, respectively, were not used because the values of $G_{xy}$ obtained under these conditions were significantly smaller than those obtained from the third and fourth modes, even when the $E_x$ value was varied (Yoshihara 2011). Except for these special cases, the $G_{xy}$ values were obtained from resonance frequencies of the first to fourth modes.

The values of $E_x$ and $G_{xy}$ obtained from the three procedures were compared, and the validity of each method was examined.

# Results and Discussion

## Young's modulus and shear modulus obtained by fitting the three analysis methods to the results predicted from FEA

Figure 3 shows the influence of the analysis method on the relationship between the Young's modulus $E_x$ obtained by fitting the results predicted from FEA and the depth/length ratio $H/L$. For the spruce and MDF models, the $E_x$ values obtained from the longitudinal vibration method using Equation 9 tended to increase as the $H/L$ ratio decreased, whereas an inverse trend regarding these values was observed for both plywood models. In contrast, no characteristic trend was found in the results obtained from the flexural vibration methods using Equations 2 and 8. Nevertheless, the variation in the Young's modulus with respect to the $H/L$ ratio is less than 5 percent, which is not more significant than that of the shear modulus, the details of which are described below.

Figure 4 shows the influence of the analysis method on the relationship between the shear modulus $G_{xy}$ fitting the results predicted from FEA and the $H/L$ ratio. Compared with the $E_x$ value, the $G_{xy}$ values, except those for the spruce model, are significantly dependent on the $H/L$ ratio. As demonstrated in previous articles (Yoshihara 2011, 2012a), the $G_{xy}$ value of the MDF model tends to increase as the $H/L$ ratio decreases, whereas those of both plywood models

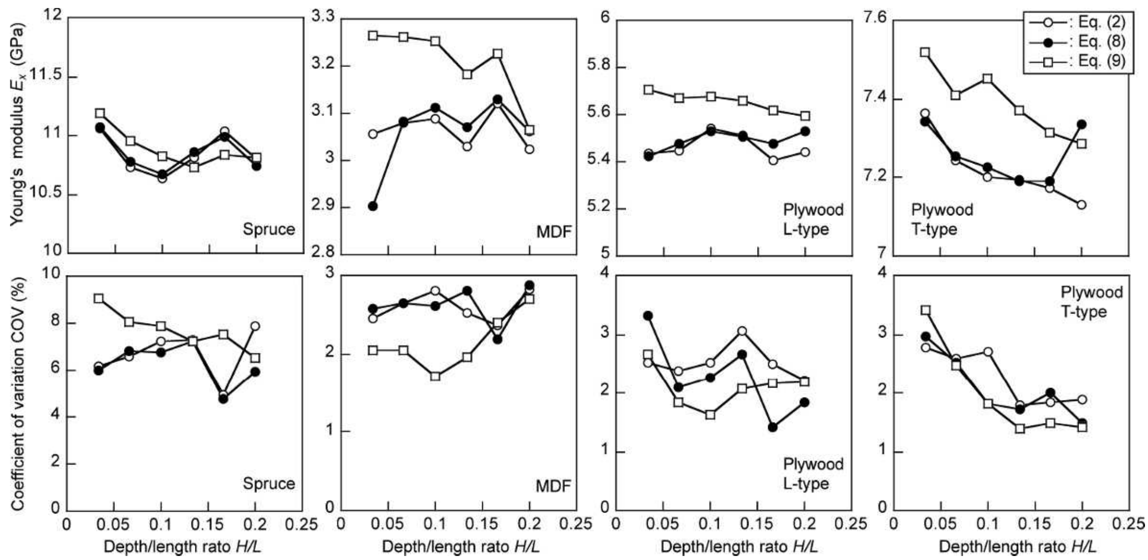


*Figure 5.—Influence of the analysis methods on the relationship between the Young's modulus* $E_x$ *obtained from the actual vibration tests and depth/length ratio* H/L. *MDF = medium-density fiberboard.*

show an inverse tendency, with only a few exceptions. Nevertheless, it is difficult to determine any significant differences among the $G_{xy}$–$H/L$ relationships obtained from the three analysis methods.

## Young's modulus and shear modulus obtained by actual vibration tests

As described above, the FEA results suggest that the analysis method examined in this study does not significantly influence the measured Young's modulus and shear modulus values. Nevertheless, the actual vibration test results are often different from those predicted by FEA.

Figure 5 shows the values of the Young's modulus $E_x$ and COV obtained from the actual vibration tests. For the spruce specimens, the influence of the $H/L$ ratio on the $E_x$ value is not so particularly significant. In addition, there are small differences between the $E_x$ values obtained from the flexural vibration analyses using Equations 2 and 8 and longitudinal vibration analysis using Equation 9. For the MDF and plywood specimens, however, the $E_x$ value obtained using Equation 9 was often larger than the values obtained using Equations 2 and 8. A statistical analysis of the difference between the $E_x$ values of specimens with different analysis methods showed that the $E_x$ value of the MDF specimen obtained using Equation 9 was significantly larger than the values obtained using Equations 2 and 8 when the $H/L$ ratio was smaller than 0.166 (corresponding to a depth of 50 mm) because the probability value ($P$ value) obtained by comparison with the corresponding $E_x$ values was smaller than a significance level of 0.01. For the L- and T-type plywood specimens, the statistical analysis showed that the difference was significant when the $H/L$ ratio was smaller than 0.133 (corresponding to a depth of 40 mm) for the L-type plywood specimens, whereas the difference was significant when the $H/L$ ratio was 0.133 and 0.1 (corresponding to depth of 40 and 30 mm, respectively) for the T-type plywood specimens.

Figure 6 shows the values of the shear modulus $G_{xy}$ and COV obtained from the actual vibration tests. In the flexural loading, the deflection caused by the shearing force is significantly smaller than that caused by the bending moment. Therefore, the measurement error tends to be included in the shear modulus, which is calculated by extracting the shear deflection, more significantly than that of Young's modulus. This trend is more pronounced with decreasing the depth/length ratio, so the COV values for the shear modulus are larger than those of the Young's modulus. As described above, the $G_{xy}$ values derived using Equations 2 and 10 were calculated by averaging those corresponding to each vibration mode. Similar to the trend observed for the Young's modulus, the influence of the $H/L$ ratio and analysis method on the $G_{xy}$ value for the spruce specimens is not particularly significant. For the MDF specimens, the $G_{xy}$ value tends to increase as the $H/L$ ratio decreases. Nevertheless, a negative value of $G_{xy}$ was obtained using Equation 8 at an $H/L$ ratio of 0.033 (corresponding to a depth of 10 mm). The negative value of $G_{xy}$ is due to the small contribution of shear deflection with respect to the bending deflection under this condition (Yoshihara 2011). In contrast, the $G_{xy}$ value of plywood specimens tends to decrease as the $H/L$ ratio decreases. These decreasing and increasing trends of the $G_{xy}$ values obtained using Equations 2 and 8 were similar to those obtained from the experimental and numerical analyses conducted in previous studies (Yoshihara 2011, 2012a). In the analyses using Equations 2 and 8, the $H/L$ ratios at which the difference between the $G_{xy}$ values of specimens with different $H/L$ ratios were not significant fell into the ranges 0.133 to 0.2, 0.067 to 0.2, and 0.1 to 0.2 (corresponding to depths of 40 to 60, 20 to 60, and 30 to 60 mm, respectively) for the MDF, L-type plywood, and T-type plywood specimens, respectively. In these $H/L$ ratio ranges, there were no significant differences between the $G_{xy}$ values obtained using Equations 2 and 8. Therefore, the shear modulus can be measured effectively by flexural vibration tests while reducing the influence of specimen geometry in these ranges of $H/L$. In contrast, the $G_{xy}$ value obtained using Equation 10 was often smaller than that obtained using Equations 2 and 8 when comparing the values at the same $H/L$ ratio. The statistical analysis revealed that the

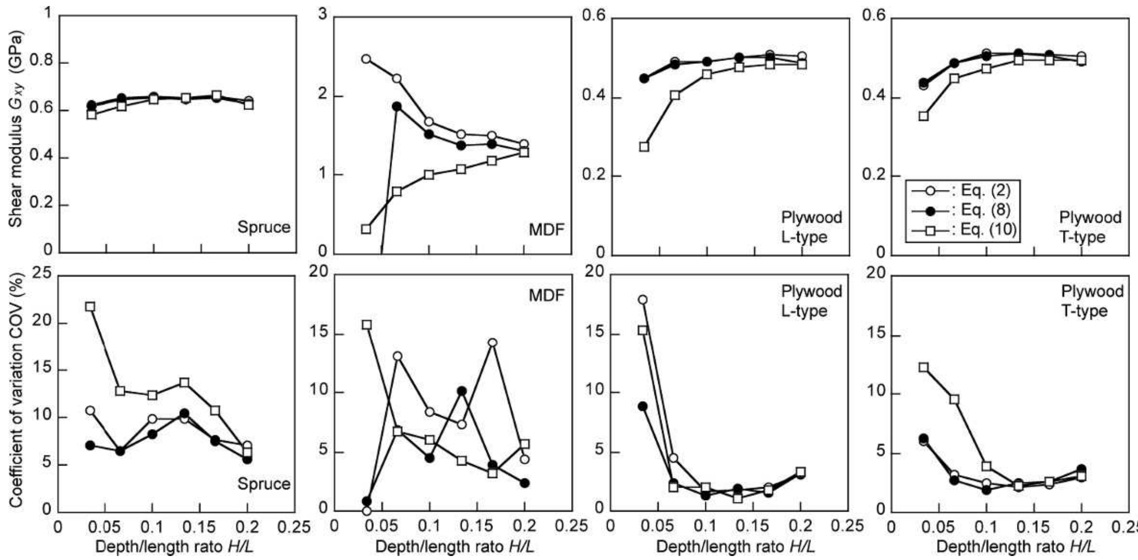


*Figure 6.—Influence of the analysis methods on the relationship between the shear modulus* $G_{xy}$ *obtained from the actual vibration tests and depth/length ratio* H/L. $G_{xy}$ *is negative at an* H/L *ratio of 0.033 (corresponding to depth of 10 mm) in the medium-density fiberboard (MDF) specimen.*

effective ranges of $H/L$ ratios required to obtain the $G_{xy}$ value while reducing the influence of specimen geometry was 0.2, 0.167 to 0.2, and 0.133 to 0.2 (corresponding to depths of 60, 50 to 60, and 40 to 60 mm, respectively) for the MDF, L-type plywood, and T-type plywood specimens, respectively. These ranges were significantly smaller than those obtained using Equations 2 and 8 shown above.

In the low-order modes of flexure, the contribution of shear deflection is relatively small with respect to that of the bending deflection. In addition, the frequencies of the low-order modes of flexure are often influenced by irregularities such as inhomogeneities within the specimen (Mead and Joannides 1991). Therefore, the shear modulus may not be precisely calculated using the data obtained from the low-order mode frequencies, particularly when combining flexural and longitudinal vibration analyses using Equation 10. To reduce this problem, it is feasible for the $G_{xy}$ values corresponding to each flexural vibration mode to be evaluated without averaging. Figure 7 shows the relationships between the $G_{xy}$ value corresponding to each flexural vibration mode and $H/L$ ratio. The dependence of $G_{xy}$ on $H/L$ is more pronounced in the first and second vibration modes than in the third and fourth modes, in which the $G_{xy}$ values coincide well with each other. When using the third and fourth modes, the effective ranges of $H/L$ for obtaining the $G_{xy}$ value while reducing the influence of the $H/L$ ratio were 0.133 to 0.2, 0.133 to 0.2, and 0.1 to 0.2 (corresponding to depth of 40 to 60, 40 to 60, and 30 to 60 mm, respectively) for the MDF, L-type plywood, and T-type plywood specimens, respectively. In these $H/L$ ratio ranges, the difference between the $G_{xy}$ values obtained from the three analysis methods was not significant. Therefore, it

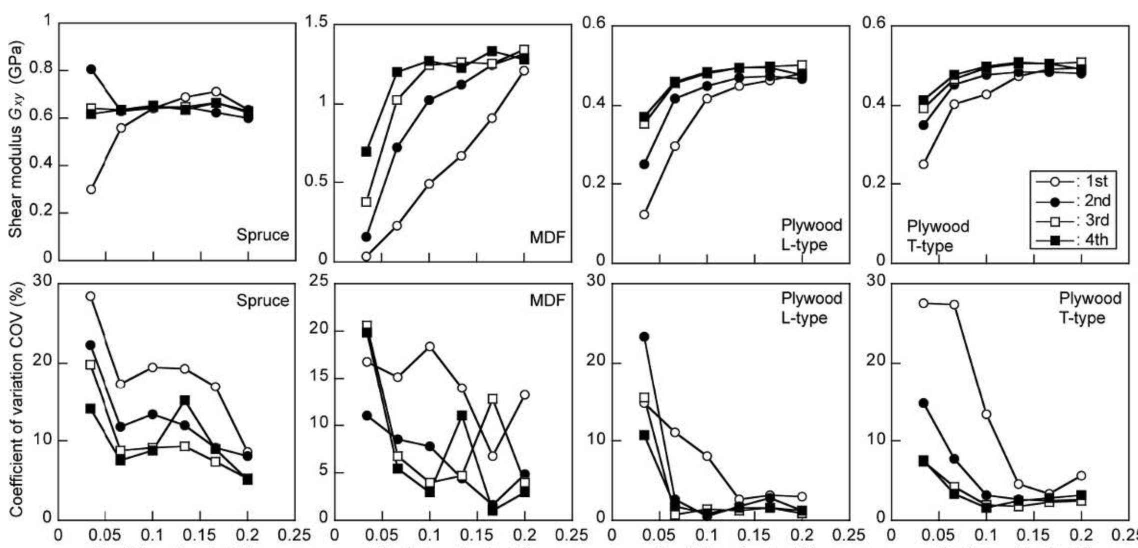


*Figure 7.—Influence of the mode number* n *and depth/length ratio* H/L *on the shear modulus* $G_{xy}$ *obtained from the combination of the flexural and longitudinal vibration analyses (Eq. 10). MDF = medium-density fiberboard.*

is recommended that the third and fourth resonance modes of flexural vibration be used to obtain the shear modulus by conducting the flexural and longitudinal vibration tests. This method does not require any mathematical software package and iteration procedure, which are required in the aforementioned methods, so it can be conducted very simply and practically.

In this study, the influence of specimen configuration and analysis method on the values of the Young's modulus and shear modulus of solid wood and wood-based materials was investigated. Nevertheless, there are several other factors influencing the measurement of these values using the vibration methods. For example, the support effect, which is not taken into account in this study, may affect the measurement of these moduli (Brancheriau and Baillères 2002). In addition, inhomogeneities due to several factors such as defects and the distribution of density may influence the measurement of these moduli (Kubojima et al. 2005, Tonosaki et al. 2010, Khademi-Eslam et al. 2011, Sohi et al. 2011). To improve the accuracy achieved in measuring the Young's modulus and shear modulus of solid wood and wood-based materials, further research is required to address the applicability of the methods examined here by conducting vibration tests using various solid-wood species and wood-based materials under various experimental conditions.

## Conclusions

Sitka spruce, MDF, and Lauan plywood samples were tested to determine their Young's modulus and shear modulus values. The flexural and longitudinal vibration tests were experimentally used, and the obtained results were analyzed by three different methods.

The FEA results indicate that the Young's modulus and shear modulus can be effectively obtained from the analysis methods examined in this study when the specimen depth is appropriately determined. Nevertheless, the experimental results indicate that the analysis method affects the values of Young's modulus and shear modulus. The values of Young's modulus and shear modulus were obtained from the analysis methods based on the rigorous solution of Timoshenko's differential equation (1921) and the iteration procedure proposed by Hearmon (1958) over a wide range of specimen depths while reducing the influence of depth/length ratio. Although the method used to obtain the shear modulus by substituting the Young's modulus measured by the longitudinal vibration test into the approximate equation proposed by Goens (1931) is simpler than the other two methods, the experimental results show this method to be inferior to the other ones when using the low-order modes of flexure in the analysis. When using the resonance frequency for flexural vibration in the third or fourth mode, however, it is promising that the Young's modulus and shear modulus can be measured by the combination of the flexural and longitudinal vibration methods while reducing the influence of depth/length ratio of the specimen similarly to the aforementioned methods. Therefore, the last method is recommended for its simplicity and practicality.

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