Determination of Crosscutting Safety Zone for Finger-Jointed *Pinus sylvestris* Furniture Components

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Abstract

A common problem with finger-jointed wooden furniture components is chipping in the finger joints due to fiber deviations around sound knots. To avoid this, a fixed size safety zone between defects and crosscuts is used, but can lead to an excess of material cut away in the crosscutting operation.

To reduce chippings in finger joints while maximizing recovery, an adaptive strategy was developed for setting the safety zone size between sound knots and finger joints in Scots pine (*Pinus sylvestris* L.) furniture components. The strategy was based upon modeling the risk of chipping the finger joint depending on the knot measurement. The model was used in an adaptive strategy in order to minimize the expected loss due to cutting away material around knots, compared with the cost of rejecting components in later stages due to chipped finger joints. Thus, each knot was assigned a unique safety zone. The strategy was tested using computer simulation of the finger-jointing process, and a sensitivity analysis was performed in order to quantify the effect of variations in the input data. The results show that the adaptive strategy improves recovery by at least 3 percent in the process of turning lumber into finger-jointed furniture components. It is very robust toward variations in knot size measurements (e.g., by scanning equipment), but less robust toward variations in crosscutting precision.

In recent years, there has been an increased interest in an integrated approach to the forestry production chain. This means that forest and tree characteristics are seen as linked to the end-user requirements for wood products (Houllier et al. 1995, Bengtsson et al. 1998, Broman et al. 2008). The aim of an integrated approach is to utilize knowledge of both the end-user requirements for different wood products and the properties of timber in order to control the flow of material from the forest to market and achieve improved use of the raw material. However, the forestry production chain is complex due to the nature of the raw material, the various stages of production, various actors, and the decisions they take in harvesting, breakdown, crosscutting, final processing, and marketing (Bengtsson et al. 1998).

This complexity leads to computer simulation being a suitable tool for analyzing the forestry production chain because it allows for experimenting with complex systems without disrupting the system itself, achieving better control over experimental conditions, and studying a system with a long time frame (Law 2007).

Current secondary processing of wood is characterized by a high degree of automation. This has increased both production speed and volume. However, discrepancies can occur between the desired and actual quality of the finished product, and consequently some products are rejected during the latter production stages, which can be a costly waste of resources. Because of the biological nature of the material, the rejection of some products due to wood features cannot be totally avoided. It is therefore important to utilize knowledge of the biological features of boards in order to handle them appropriately in the production process to minimize this loss. In the case of vertically finger-jointed furniture components, one important defect causing rejection is chipped finger joints (Broman and Fredriksson 2011). This defect occurs in milling a finger joint in which fibers at an angle to the board surface have a lower elastic strength in the vertical direction than the fibers parallel to the surface (Bodig and Jayne 1982). These fibers are thus more easily

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chipped away in milling the joint because cutting is against the grain.

Currently, a common strategy to deal with this problem is to use a safety zone between finger joints and defects, such as knots, which has the downside that a larger amount of material is cut away in the finger-jointing process. It is common practice to set the size of this safety zone to a fixed value regardless of type and/or size of the defect.

Öhman and Chernykh (2011) describe a model to predict the size of the so-called diving grain area (Fig. 1), based on knot size and location. This area is almost exclusively located around sound knots, whose fibers are intergrown with the wood fibers of the stem, compared with a dead knot, which is not connected to the normal wood in the same fashion. Their model is supposedly useful for reducing chipping in finger joints, but they do not elaborate on how this may be achieved.

Another way to approach this problem is to detect and measure the diving grain area around a knot by using, for example, laser and the tracheid effect. This type of measurement is good, but not perfect, because it is difficult in some cases to separate diving grain from other features due to surface unevenness, dirt, or compression wood, pith, or bark (Zhou and Shen 2003).

A strategy for choosing a safety zone around sound knots based on their size instead of a fixed zone has the potential to reduce chipping in finger joints, while maintaining a recovery in the finger-jointing process itself, which is as high as the one obtained when using a fixed zone. The effect of this is that the overall recovery for the production process is improved since less material is sorted out in a later stage because chipped finger joints are avoided. Such a strategy has other benefits. For example, depending on the needs of the specific component manufacturer, there may be no need for expensive laser equipment to measure diving grain area because the measurement of the size of sound knots can be achieved by using gray-scale cameras.

The decisions made in the processing of wood should be aimed at minimizing overall material loss because this is the primary source of costs in sawmills, and one of the most important budget considerations in secondary processing operations. Thus, a strategy for setting a flexible safety zone should not only reduce chipping or improve recovery in one part of the production process but also minimize the loss in the entire value chain.

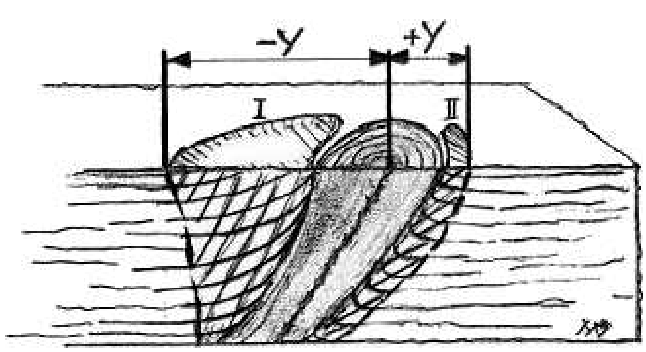

Figure 1.—Principal sketch of the diving grain area around a knot. I and II correspond to the regions surrounding the knot where the fibers deviate substantially from the longitudinal direction. Image from Öhman and Chernykh (2011).

A suitable measurement of the efficiency in a crosscutting process is length recovery, which is calculated as the length of material output from the process divided by the length of material input. This measurement does not take into account planing of the boards, as volume recovery would do, and is thus a better measurement of how good the crosscutting process is.

Based on previous research, a model of the risk of chipping relative to sound knot size and distance to a finger joint is a hypothetically beneficial approach. Since it is not certain whether a knot near a finger joint will lead to chipping, this risk must be quantified prior to modeling. Such a model can be used in a strategy in which the aim is to minimize the expected loss of material in the process through a flexible safety zone between the sound knot and finger joint. It is acceptable to quantify the outcome of the strategy in terms of length recovery.

The objective of our study was to evaluate a flexible strategy to decide the safety zone between knot and finger joint. This strategy was based on measurements of knots in finger-jointed furniture components and used sound knot size to predict the risk of chipping a finger joint. An evaluation was made by computer simulation of the crosscutting and finger-jointing process. The aim of the strategy was to improve length recovery in the production process from board to finger-jointed component compared with a fixed safety zone approach, and its robustness toward variations in input data was assessed and quantified.

Materials and Methods

The study was based on wood furniture components, made from 177 Scots pine (*Pinus sylvestris* L.) logs. The studied components were a type currently produced in the Swedish wood industry, finger-jointed bedsides with final product dimensions of 25 by 110 by 2,018 mm. The logs were taken from different parts of the tree, i.e., butt, intermediate, and top logs, and from the same geographical region in Sweden between latitudes of 64°N and 66°N. The logs had a top diameter ranging between 134 and 147 mm. They were each sawn into two 33 by 120-mm center boards and kiln dried to 14 percent moisture content. The nominal dimensions after drying were 31 by 115 mm with varying lengths.

The secondary processing is described by a flow chart in Figure 2. The boards were scanned using a WoodEye (Innovativ vision AB 2012) industrial scanner that was equipped with four gray-scale line cameras for defect detection and a laser for measuring board dimensions and grain direction. Unwanted defects such as large knots, wane, or cracks were cut away from the boards, and the resulting pieces finger jointed together and crosscut to a final length

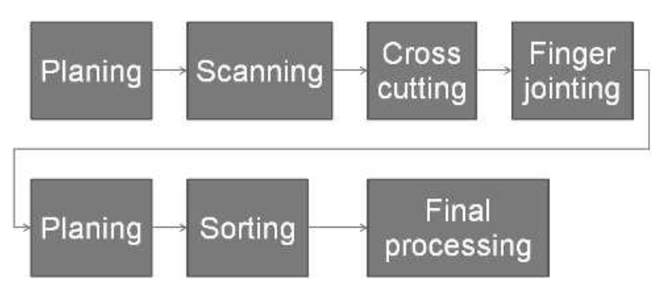

Figure 2.—Description of the furniture production process, from boards to furniture components.

of 2,018 mm. In the crosscutting operation, a safety zone of 10 mm from the crosscut positions to all defects was used in order to avoid chipping in the finger joints and to account for errors in cutting position. No crosscutting in any defects was allowed, regardless of whether they had been classified as accepted or rejected. A planing operation took place both during and after the finger-jointing process, planing the boards first to a 30 by 114-mm cross section and then to the final dimensions of 25 by 110 mm. During the final steps of the production process, the boards went through a quality inspection in which defect components were sorted out before the final processing of the product.

In the final product, data on knots in the proximity of the finger joints were collected by using manual measurements on all components, regardless of whether they had been judged as acceptable in the quality inspection.

Measurement of knots

All sound knots within 30 mm of the tip of the teeth of each finger joint were measured using a carpenter's rule. However, knots with a diameter smaller than 2 mm in all directions were disregarded. The measurement principle is described in Figure 3. The following features were measured and registered (the unit of measurement in square brackets):

- D = Distance from finger joint to sound knot (from tip of tooth to knot edge, lengthwise direction of board). This was set to 0 if the knot was within the finger joint [mm].
- L = Length of sound knot (lengthwise direction of board) [mm].
- Whether the finger joint is chipped or not [yes, no].

Overall, 1,173 knots were measured. Some types of knots, such as splay knots, were disregarded because they do not cause any damage to the finger joints in this material, i.e., they were measured, but not included in the final analysis. Furthermore, splay knots account for less than 10 percent of the total knot population in this material. Dead knots and bark-ringed knots were also disregarded because the diving grain area around these knots is very small or nonexistent (Tyvand 1991, Buksnowitz et al. 2010).

Figure 4 shows an example of a chipped finger joint within the diving grain area of a sound knot.

In Figure 5, some examples of the types of sound knots that were measured are presented.

Data analysis

From the measured knot data, a linear regression model was constructed to predict the risk of chipping in the finger joint depending on two variables: size of sound knot in the longitudinal direction (L) and distance between knot and finger joint (D). A prerequisite for this was that the variables were both normally distributed and independent. However, the variable L was not normally distributed but followed a log-normal distribution. Therefore, the logarithm of base 10 of the knot length was chosen as a predictor for the finger-joint chipping risk.

In order to calculate the risk of chipping, the knots were divided into 16 classes based on L and D, according to Table 1, each class having between 12 and 312 knots. The risk of chipping for each class was calculated as the number of knots causing chipping divided by the number of knots.

For fitting the prediction model, the class center values were used for both L and D: (class upper limit + class lower

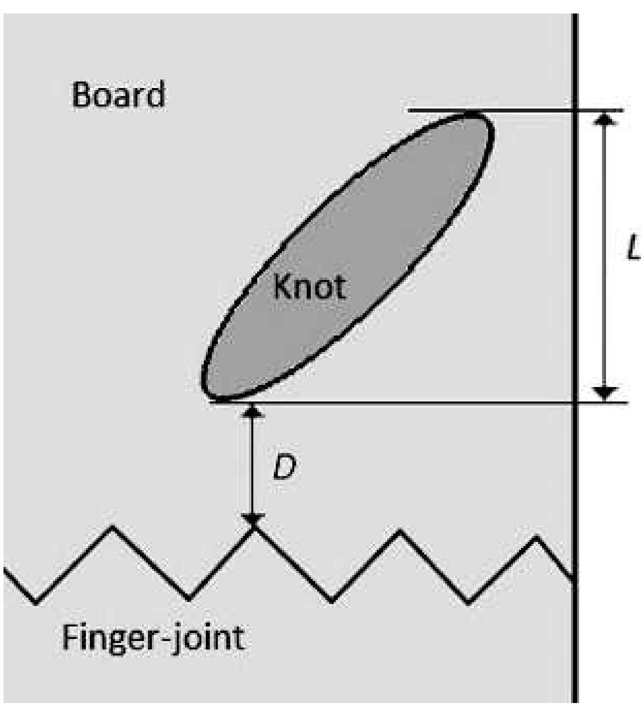

Figure 3.—Principle of measurement. L = *length of a sound knot in the lengthwise direction of a board,* D = *distance from a finger joint to a sound knot.*

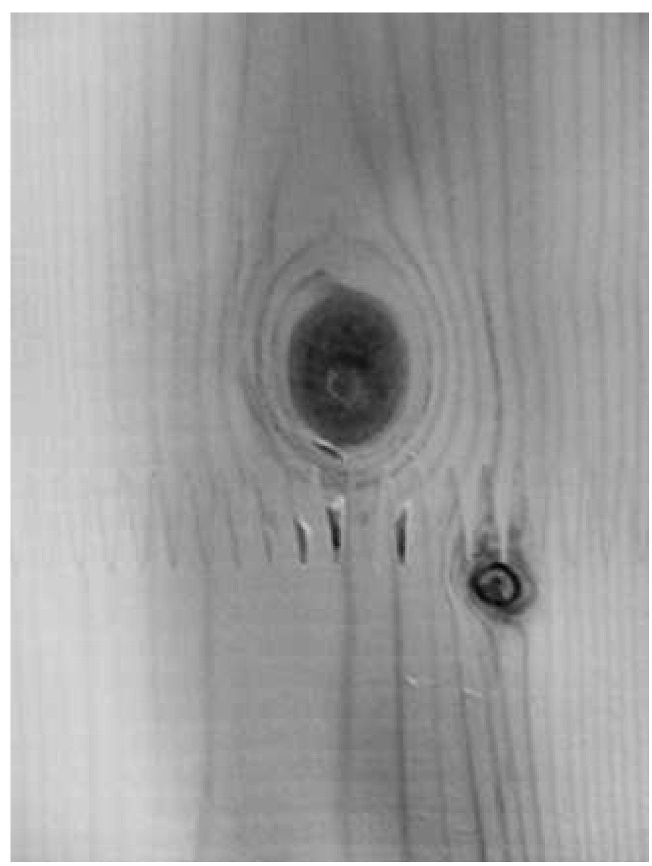

Figure 4.—Typical damage to a finger joint due to the proximity of a sound knot. Note that the area around the dead knot is not affected.

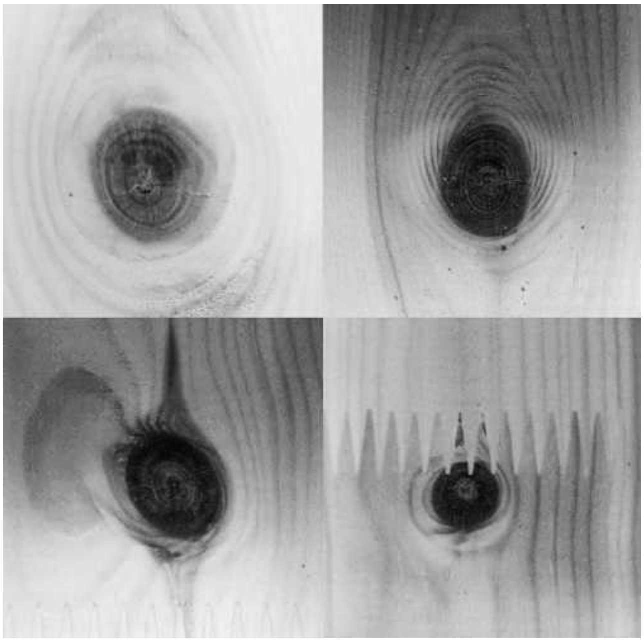

Figure 5.—Typical examples of the types of knots that were measured.

limit)/2. For the classes containing all knots above a certain size and/or distance to a finger joint, the lower limit plus one-half of an interval width was chosen.

Furthermore, the risk was strictly decreasing toward 0 with increasing distance from the sound knot and decreasing size of the knot, and thereafter continued at a constant 0. Thus, only values of risk above 0 were used for the linear model, and cases in which modeled risk was predicted to be below 0 were set to 0. This is shown schematically in Figure 6.

New safety zone strategy

Based on the results from the analysis of the knot measurements, a strategy for deciding the safety zone length between sound knots and finger joint was developed.

The safety zone was decided for each sound knot in order to minimize the expected loss resulting from the decision. This was based on game theory and decision making under uncertainty, albeit in one of its simplest forms. The expected length loss (E_L) was calculated according to

$$E_L = \text{risk}_{\text{fj}D} \cdot \text{length}_{\text{comp}} + D \quad (1)$$

where

$\text{risk}_{\text{fj}D}$ = risk of chipped finger joint, between 0 and 1, modeled from knot size;

Table 1.—Classification of sound knots with regard to knot length (L) and distance to finger joint (D).

	L (mm)			
D (mm)	0–7	8–15	16–23	24+
0–2	Class 1	Class 2	Class 3	Class 4
3–5	Class 5	Class 6	Class 7	Class 8
6–8	Class 9	Class 10	Class 11	Class 12
9+	Class 13	Class 14	Class 15	Class 16

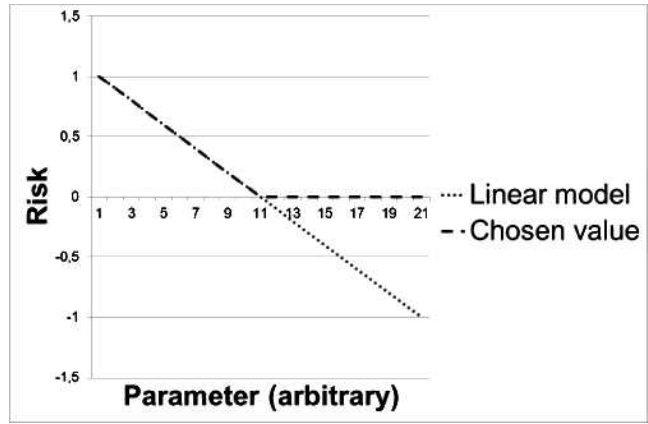

Figure 6.—Principle of using a linear model to predict the risk of chipping a finger joint due to a sound knot. When the model predicts values below 0, the risk is set to 0. Note that this is not the actual model, but an illustration of the principle.

$\text{length}_{\text{comp}}$ = length of finger-jointed component (mm; in this case 2,018 mm); and

D = safety zone length (mm; Fig. 3).

The variable $\text{risk}_{\text{fj}D}$ was calculated using the linear regression model relating $\text{risk}_{\text{fj}D}$ to L and D, and because $\text{risk}_{\text{fj}D}$ depends on D, D values between 0.1 and 15 cm were tested for each sound knot. The distance that resulted in the lowest expected loss was chosen for that particular knot.

The loss function was calculated one knot at a time. Since one chipped finger joint resulting from one knot is enough to cause rejection of an entire component, risk multiplied by the entire component length is one term of the expected loss function. The other term is set to the safety zone distance since this ''risk'' is exactly 1. Possible rejection caused by several knots in a component is not accounted for; however, this is a rare case because each component contains four to five finger joints and only a small percentage of joints have knots nearby.

Simulation

The new strategy was tested using a computer simulation program described by Fredriksson (2011). This software uses data from industrial scanners in order to predict the outcome of a crosscutting and finger-jointing operation on boards. Defects such as sound knots, dead knots, cracks, and wane, are represented. The output from the program includes length recovery, which is defined as length of finger-jointed components divided by length of boards, and average length of the crosscut pieces. For this application, a module for calculating the chipping risk according to the model above was added. The input data to the simulation described in this article were the results from scanning the boards, with all identified defects included.

The overall length recovery was calculated as the length recovery in the finger-jointing process multiplied by $1 - \text{risk}_{\text{fj}D}$. It was assumed that each chipped finger joint leads to one rejected component, which was reasonable because each 2,018-mm finger-jointed component usually contained around four to five joints, and only a small percentage of the joints had sound knots nearby.

Several cases of a fixed safety zone were tested through simulation and compared with the strategy of using a safety zone based on sound knot size.

Sensitivity analysis

In a practical situation, there is variation both in the accuracy of the scanning of the boards and in the crosscutting operation. However, the distribution of this variability is not known, and until data are available it is reasonable to assume that variability is normally distributed since the errors in a calibrated industrial system should not be unevenly distributed around the mean. The measurements made in this study indicate this, as shown in the ''Results'' section. These measurements were made from knot to finger joint and did not reveal whether the source of variation was the scanning or the crosscutting equipment. Thus, they were not considered to represent the correct distribution of either, but as an indicator of normal distribution.

In order to assess how this variability affected the model and the strategy, several simulation runs were made. In these simulations the sound knot length L and the safety zone were given random errors according to normally distributed functions with varying standard deviations and expected value 0. This was done in order to find the level of uncertainty regarding these two values for which the strategy provides disadvantageous results in terms of overall length recovery. Three different standard deviations were used for both length and safety zone, with all combinations of those setups, for a total of nine runs. The same seed was used for the randomizing algorithm each time, according to the method of common random numbers (Law 2007).

Results

Measurement of knots

The distributions of the measured variables are presented for sound knot length (L) in Figure 7 and for the distance between sound knot and finger joint (D) in Figure 8. Figure 9 presents the distribution of a logarithmic transformation of L.

Model

The linear model for predicting risk of chipping in a finger joint, with regard to nearby sound knots, is

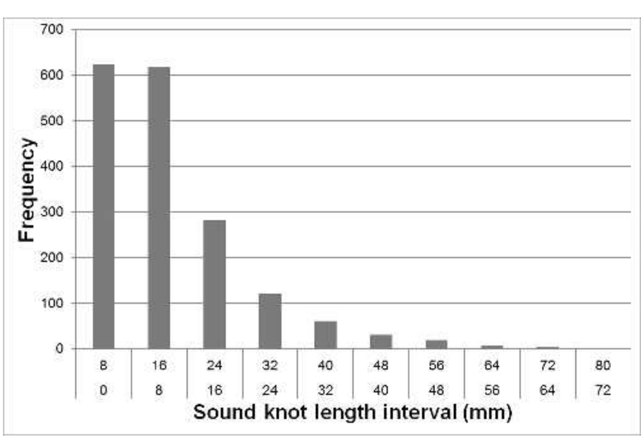

Figure 7.—Distribution of the sound knot length or variable L.

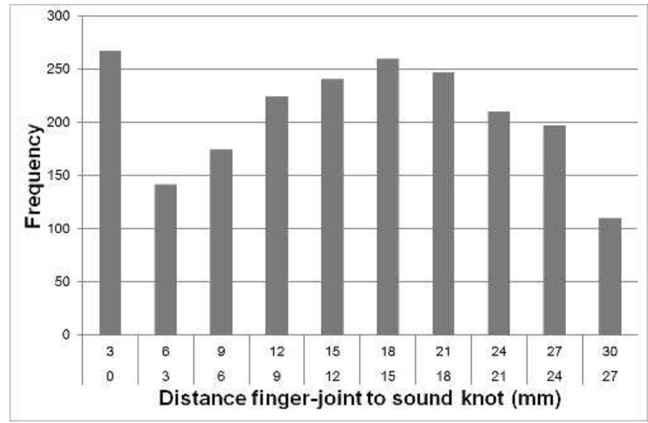

Figure 8.—Distribution of the distance between a sound knot and finger joint or variable D. *Note: The first interval contains all knots within or very near the finger joint. Thus, this is a summary of the left tail of the distribution curve, and is an effect of not using a "negative distance" in the measurement principle.*

$$\text{risk}_{\text{fj}D} = -0.323 + 0.771 \cdot \log(L) - 0.0752 \cdot D \qquad (2)$$

where

L = length of sound knot in the board's lengthwise direction (mm), and

D = distance from the finger joint to the knot (mm).

When applying the model on the 16 classes of knots and setting all negative results to 0, this model yields a root mean square error (RMSE) of 0.067 and a calculated R^2 of 0.92, when comparing the observed risk of chipping within each class with the predicted risk. The calculated and measured risks of each class are shown in Table 2. In this prediction, the class center values for L and D were used. For each class, RMSE was calculated as the square root of the mean of the square errors in the class, which in turn was calculated as $(\text{risk}_{\text{obs}} - \text{risk}_{\text{pred}})^2$, where risk_{obs} is the observed risk for the class, and $\text{risk}_{\text{pred}}$ is the predicted risk for the class. R^2 was calculated as $1 - SS_{\text{err}}/SS_{\text{tot}}$, where SS_{err} is the sum of square of residuals of the model, and SS_{tot} is the sum of total squares for the population.

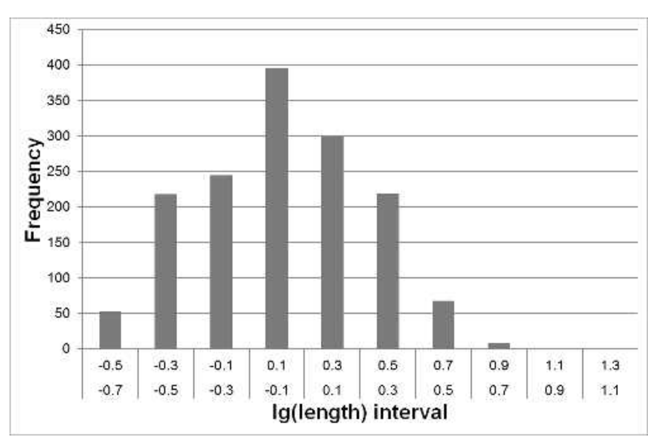

Figure 9.—Distribution of the sound knot length or variable L, *after logarithmic transformation.*

Table 2.—Modeled and measured risk for all classes of knots used in this study.[a]

	L (mm)							
	0–7		8–15		16–23		24+	
D (mm)	M	R	M	R	M	R	M	R
0–2	0.03	0.02	0.40	0.44	0.56	0.72	0.68	0.73
3–5	0.00	0.00	0.17	0.09	0.34	0.19	0.45	0.33
6–8	0.00	0.00	0.00	0.00	0.12	0.11	0.23	0.21
9+	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02

[a] D = distance; L = length; M = modeled risk; R = ''real'' or measured risk.

Table 3.—Strategy of using a flexible safety distance based on knot size compared with using several values for a fixed distance.

Strategy	Overall recovery (%)	Recovery, finger-joint process (%)	Avg. piece length (mm)	Chipping risk (%)
Fixed distance (mm)				
10	70.7	83.1	533.9	14.9
20	79.9	80.8	531.2	1.2
30	78.3	78.3	524.5	0.0
Flexible distance	83.1	83.1	533.7	0.0

Table 4.—Results when using different random variations in knot size and safety zone size.[a]

Knot size, σ (mm)	Safety distance, σ (mm)	Overall recovery (%)	Recovery, finger-joint process (%)	Avg. piece length (mm)	Chipping risk (%)
10	10	79.7	81.2	531.5	1.9
15	10	79.4	81.1	529.6	2.1
20	10	79.9	81.2	530.0	1.6
10	15	77.3	79.7	527.9	3.0
15	15	76.2	79.0	529.0	3.6
20	15	77.4	79.4	526.4	2.5
10	20	73.1	76.3	525.8	4.2
15	20	72.7	76.9	522.8	5.5
20	20	71.9	76.3	523.9	5.8
Reference[b]	20	79.9	80.8	531.2	1.2

[a] σ = standard deviation.
[b] Reference = results for using a fixed safety distance of 20 mm; see Table 3.

Simulation

The results of the simulation using the two strategies are shown in Table 3 as overall length recovery, recovery for the finger-jointing process alone, average piece length, and average risk of chipping finger joints. Average piece length is the length of the crosscut pieces that are to be joined, which should be as long as possible for an efficient finger-jointing process.

Sensitivity analysis

The results of the simulations using different errors of the sound knot length and calculated distance are presented in Table 4. The fixed zone strategy with the highest overall recovery from Table 3 (20 mm) is included as a reference. In the reference the sound knot length L and cutting position were not varied.

Discussion

We have shown that it is possible to model the risk of chipping in finger joints and account for this risk in a strategy for crosscutting and finger jointing in a robust way by using a flexible safety zone between sound knots and finger joints. This is an approach that promises higher recovery for a finger-joint production process than a strategy using a fixed safety zone. It has other benefits, such as only needing information from gray-scale cameras, thus reducing the need for expensive laser equipment and increasing the average crosscut piece length in the finger-jointing process, which reduces the production cost per unit length of finger-jointed component. However, there are some limitations and issues regarding the strategy's applicability.

The boards that the measurements were made on were planed (2.5 to 3 mm on each side) in contrast to the material scanned and thus used for the simulations. This means that the defects on the simulated boards did not have exactly the same dimensions as the physical boards. However, the size of a knot was not changed by more than 1 or 2 mm in most cases by this level of planing, and thus there was not a substantial impact on the results, as shown in the sensitivity analysis.

Some knots were excluded in the measurements, including any knots smaller than 2 mm and splay knots. This is not a significant issue because knots smaller than 2 mm have a very small impact on recovery in this type of production process, and the splay knots, at least in this study, account for less than 10 percent of the total knot population near finger joints.

The material in this study was limited in size, and before any general conclusions can be drawn, tests need to be made on other material as well. It was also limited by species, i.e., to Scots pine, and the conclusions should not be applied to other species. Furthermore, the material was geographically limited to northern Sweden, where trees are rather slow growing. The fiber deviation area around a knot is a little different in faster growing trees. For instance, the size of growth rings depends upon the growth rate of the tree, and the growth ring size affects the strength of the wood material, and thus the risk of chipping. The material was limited to boards from logs with a certain range of diameters, but from different types of logs. Additional tests using a different material would strengthen the conclusions of this study.

The model was based on measurements of knots that were not cut away. This means that sound knots longer than 45 mm were not included in the model because this was the quality threshold for sound knots in the studied production process. It is not certain that extrapolating the model to these sizes would be correct, and this needs to be tested as well. The loss function did not take into account the possibility of two or more knots causing rejection of a component. This is not a common case, but it would be prudent to add this in an improved future model.

The strategy minimized risk. This was due in a large degree to the final product length of 2,018 mm. Because the product was relatively long, the risk of chipping a finger joint and thus rejecting an entire component made it worthwhile to cut off a little extra material, usually a few millimeters, in order to reduce the risk to near 0. For a shorter component length, the strategy would probably be more risk prone, as the potential recovery loss of sorting out rejected components is smaller.

In a practical application, the actual cutting position and the desired cutting position do not always match. This needs to be accounted for by a cutting offset, which would need to be included in the safety zone calculation. However, it does not interfere with the general idea behind the strategy.

As with most computer simulations, any predictions regarding the actual outcome of a certain strategy are difficult to make. In this study, two different cases were compared, and it can be concluded that one strategy was better than the other in terms of overall length recovery. Thus, no predictions regarding the actual length recovery were made, merely that it can be improved by a flexible safety zone strategy.

The proposed strategy was robust to variations in the input data of measured sound knot length and cutting position, which points to applicability in real situations. It was most sensitive to variations in the actual cutting position, for which a standard deviation of more than 10 mm will drastically reduce recovery with regard to the desired position. This needs to be accounted for when applying this strategy practically by ensuring that the actual cutting position does not deviate from the desired cutting position with a larger variance. Whether this is actually possible was not addressed here.

The conclusion is that recovery in a finger-joint production process is increased by at least 3 percent, as shown in Table 3, when using a flexible strategy for creating a safety zone around sound knots. This is the difference between the best performing fixed zone strategy and the flexible strategy, but it is even higher when compared with the situation of the finger-jointing process used as reference in this study.

The strategy is based on a model for predicting the risk of chipping in a finger joint, depending on distance to and size of sound knots near the joint, and offers significant advantages over using a fixed safety distance in terms of recovery. It is robust toward variations in input data up to a standard deviation of 10 mm for cutting position and up to at least 20 mm for knot size.

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