Length Effect on the Tension Strength between Mechanically Graded High- and Low-Grade Chinese Fir Lumber

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Abstract

An experimental study was conducted to evaluate the length effect on the parallel-to-grain tensile strength of Chinese fir (*Cunninghamia lanceolata*) lumber. In all, 473 pieces of mechanically graded lumber were tested at gauge lengths of 150, 200, 250, and 300 cm. The lumber was sorted into matched groups according to the dynamic Young's modulus as measured using the longitudinal vibration method. The averages of the dynamic Young's modulus of high-grade (H) and low-grade (L) specimens were 11.8 and 8.9 GPa, respectively. Using nonparametric estimates, the estimated length effect parameters of H and L were 0.188 and 0.226, respectively, for the 50th percentile and 0.185 and 0.318, respectively, for the 5th percentile. It was then concluded that the different length effect factors between H and L could be used when using the lumber for practical purposes. The effect of increasing length on the tensile strength was a little larger in H but smaller in L for the 50th percentile compared with the 5th percentile. When two-parameter Weibull distribution functions were fitted to the strength data, the estimated shape parameters of the Weibull distribution by the parametric method were almost identical to the inverse of nonparametric parameters. The influence of defects such as knots on the lower tail of the strength distribution in H may be different than that in L.

The design of structures using I-joists, wood trusses, beams, etc., may be governed by the strength of the structural dimension lumber in tension parallel to the grain. In the past, the tensile strength properties of dimension lumber were derived by testing small, clear specimens with adjustments for defects such as knots and the slope of the grain. A more recent approach to the development of tensile strength properties involves testing full-size lumber. ASTM D1990 (American Society for Testing and Materials 2000) indicates that the property values of all lumber test data should be adjusted to the characteristic size, and the length effect factor on tensile strength is 0.14 expressed exponentially. In China, tensile test methods for dimension lumber are still being drafted. Because no existing tensile strength data are available for Chinese species, the method of length adjustments of ultimate tensile strength (UTS) may not be included in the draft. This study assessed the length effect in tension of Chinese fir based on tension test results and appropriate data analyses.

The model to quantify the effect of size was based on the weakest link theory. Bohannan (1966) reported the first study in which the Weibull brittle fracture theory was applied to wood. He studied clear wood beams and found that for geometrically similar beams (i.e., beams with the same span-to-depth ratio and loading configuration), the strength was proportional to the depth of the beam to the power 1/9, this being the result of a depth effect and a length effect of equal importance. Many studies have used the weakest link theory—for example, the tension strength perpendicular to the grain of Douglas-fir (*Pseudotsuga menziesii*) reported by Barrett (1974), the length effects in 38-mm spruce-pine-fir (SPF) dimension lumber reported by Madsen (1990), the effect of length on the tensile strength of visually graded kiln-dried nominal 2 by 4-inch SPF lumber reported by Lam and Varoglu (1990), and a comparison of the length effect models for lumber tensile strength reported by Taylor et al. (1992).

Evidence supports the need to study the tension length effects in mechanically graded Chinese fir (*Cunninghamia lanceolata*). Madsen and Buchanan (1986) showed species dependence based on bending test results. It was also thought that the length effect in mechanically graded lumber

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would be different from the length effect in visually graded lumber, a theory studied by many researchers. Therefore, the length effect might be dependent on the mechanical grade of the lumber, because characteristics such as numbers and sizes of knots likely are different for each grade. In this project, the differences in the length effect between mechanically graded high-grade and low-grade Chinese fir lumber were studied and compared.

Theory

A statistical strength theory has been developed on the basis of the weakest link theory, which states that when subjected to tension a chain is as strong as its weakest link. The size effect (Madsen 1992) on the strength of lumber is based on the weakest link theory. The length effect on tensile strength using the brittle fracture theory is described as a relation between the length and strength of two members with the same cross-sectional shape and different lengths. The relation is given by

$$\frac{x_1}{x_2} = \left(\frac{L_2}{L_1}\right)^s = \left(\frac{L_1}{L_2}\right)^{-s} \tag{1}$$

where x_1 and x_2 are the strengths of members of length L_1 and L_2, respectively, and s is the length effect parameter. The change in strength for doubling the length can be obtained by setting $L_2/L_1 = 0.5$. If s becomes greater, the effect of doubling the length becomes severe, and for $s = 0.3$, only 81 percent of the strength remains.

Suppose that each member consists of a large number of statistically independent brittle elements selected at random from a parent population of elements with a cumulative distribution function of strength given by a three-parameter Weibull (3P-Weibull):

$$F(x) = 1 - \exp\left[-\left(\frac{x - x_0}{m}\right)^k\right] \tag{2}$$

where x is the strength and k, m, and x_0 are parameters of the 3P-Weibull (k is the shape parameter, m is the scale parameter, and x_0 is the location parameter).

If the location parameter x_0 is assumed to be zero, as is often done, the 3P-Weibull described above reduces to a two-parameter Weibull (2P-Weibull) with the parameters k and m. The 2P-Weibull is given as

$$F(x) = 1 - \exp\left[-\left(\frac{x}{m}\right)^k\right] \tag{3}$$

where x is the strength, k is a shape parameter, and m is a scale parameter.

If a member contains n elements, the cumulative distribution function of this member should be derived from the function of one element. When the function of one element can be assumed in the 2P-Weibull, the function with n elements is given as

$$1 - F_n(x) = \{1 - F_1(x)\}^n = \exp\left[-n\left(\frac{x}{m}\right)^k\right] \tag{4}$$

where x is a strength and $F_n(x)$ and $F_1(x)$ are the 2P-Weibull of n elements and one element, respectively. Equation 4 can be rearranged to give the strength at any quantile q in the distribution:

$$x_q = mn^{-1/k}[-\ln(1-q)]^{1/k} \tag{5}$$

Now consider two members of different sizes containing n_1 and n_2 elements. In this case, the ratio of strength of two sizes at any quantile q is

$$\frac{x_q(n_1)}{x_q(n_2)} = \frac{mn_1^{-1/k}[-\ln(1-q)]^{1/k}}{mn_2^{-1/k}[-\ln(1-q)]^{1/k}} = \left(\frac{n_1}{n_2}\right)^{-1/k} \tag{6}$$

When the distribution of the strength follows 2P-Weibull, s in Equation 1 and $1/k$ in Equation 5 are the same value at any quantile q.

In general, there are three methods to obtain estimates of the size effects from experiments: (1) the slope method, (2) the shape parameters, and (3) the fracture position shown by Madsen (1992). Because the last method is applicable to a simply supported beam with a concentrated load in the center of the span, it was not considered here. With the slope method, Equation 1 can be rearranged to give a linear relation between the logarithm of strength and the logarithm of length:

$$\frac{\ln x_2 - \ln x_1}{\ln L_2 - \ln L_1} = -s \tag{7}$$

where s is the length effect parameter, which is the slope of the regression line of x on L (disregarding the negative sign). With the shape parameter method, s is the inverse of k of the 2P-Weibull presented in Equation 6. This method is generally used for estimating not only the length effect parameter but also the depth, width, and volume effect parameters.

The 50th and 5th percentiles of tensile strength distributions were obtained by the nonparametric method according to ASTM Standard D2915-03 (ASTM International 2003). The sample nonparametric percent point estimate (NPE) at any quantile q is given by

$$\text{NPE} = [q(n+1) - (j-1)](x_j - x_{(j-1)}) + x_{(j-1)} \tag{8}$$

where x_j is the jth value by arranging the test values in ascending order, n is the sample size, and j is the smallest order satisfying $j/(n+1) \geq q$. In the following section, NPM and NPL, which denote 50th and 5th percentiles, respectively, estimated by the nonparametric method are used.

Materials and Methods

Materials

Unlike in North America, there is currently no commercial production from Chinese fir plantations in China. The sampling plan focused on collecting representative logs from regional plantation stands rather than lumber from sawmills. Chinese fir lumber was sampled at a forestry center in Sanming city of Fujian province in China. A total of 220 standing trees of plantation-grown Chinese fir with diameters at breast height (DBH) ranging from 250 to 320 mm were collected from Sanming in Fujian province for the preliminary in-grade lumber testing program. Trees from the DBH range sampled are expected to produce logs with less juvenile wood, higher-density wood, and greater mechanical properties. The number of trees was roughly in proportion to the stock volumes in the region.

Lumber was sawn from the logs following a pattern typically used in China to maximize the volume of

Table 1.—Dimension, test span, and properties of specimens.[a]

Specimen	n	Width (cm)	Height (cm)	Length (cm)	Test span (cm)	Annual ring width (mm)	Density at 12% (g/cm^3)	E_f (GPa)[b]
Grade H								
H150	59	9.0	4.5	370	150	4.6 (39.0)	0.371 (11.8)	11.86 (11.5)
H200	59	9.0	4.5	370	200	4.9 (37.2)	0.366 (12.6)	11.72 (11.7)
H250	60	9.0	4.5	370	250	4.2 (37.9)	0.378 (11.4)	11.81 (10.8)
H300	59	9.0	4.5	370	300	5.5 (36.5)	0.371 (12.3)	11.69 (10.1)
Grade L								
L150	59	9.0	4.5	370	150	5.6 (35.5)	0.350 (11.3)	8.86 (9.7)
L200	59	9.0	4.5	370	200	6.0 (36.2)	0.338 (11.3)	8.87 (9.4)
L250	59	9.0	4.5	370	250	7.2 (35.4)	0.337 (12.9)	8.90 (9.4)
L300	59	9.0	4.5	370	300	7.0 (40.0)	0.339 (9.7)	8.86 (9.1)

[a] Values in parentheses are coefficients of variation (%). H = high-grade lumber; L = low-grade lumber.
[b] E_f = Young's modulus measured by the longitudinal vibration method.

recovered sawn timbers. The sawn lumber was nominally 5.0 cm thick, 10.0 cm wide, and 370 cm long. After kiln drying, these rough-sawn boards were selected with a Chinese-made, continuous mechanical grading machine. The machine measures the localized flat-wise Young's modulus for each lumber specimen and calculates the average of the measured values within a specimen. The target average values of Young's modulus of the two sampled groups were 8 and 11 GPa, respectively. In the following section, the former group (low-grade lumber) is called the L specimen, and the latter group (high-grade lumber) is called the H specimen. Because the lumber was not planed, the actual values of Young's modulus were higher than the values indicated by the machine. After grouping, all sawn lumber was finally planed to dimension lumbers with a cross-section size of 4.5 by 9.0 cm. The length remained the same (370 cm).

The dynamic Young's modulus (E_f) values of the selected dimension lumber were measured by the longitudinal vibration method. They were calculated from the resonance frequency determined from a fast Fourier transform spectrum analysis of the tap tone. The specimens were ranked according to their E_f values in ascending order. In the case of the H specimen, lumber with the four lowest E_f values was selected. The lumber was assigned according to the ascending order among these four E_f values. The one with the smallest value was assigned to the H300 group (lumber grade, high; gauge length, 300 cm), followed by the H250, H200, and H150 groups. The specimens with the next four lowest E_f values were then selected and assigned similarly. This process was repeated until all the lumber was assigned to the four groups. A similar process was followed for creating the samples of L specimens. The dimensions, annual ring width (ARW), density, and E_f were measured for each specimen. All samples were stored in a conditioning chamber maintained at 20°C and 65 percent relative humidity.

Tensile test

Tensile tests were conducted using a tensile test machine (Metriguard Model 401). The tension machine was equipped with serrated plates to grip the specimens. Test spans (gauge length) were 150, 200, 250, and 300 cm, as shown in Table 1. The average moisture content (MC) measured at the failure location by the ovendry method was 12 percent, with a small standard deviation (0.5%). Twelve percent is the MC value at which lumber design properties will be derived in China. Test time to failure ranged from 3 to 5 minutes.

Results and Discussion

Characteristics of specimens and distribution of tensile strength

Table 1 shows the sample sizes, dimensions, ARWs, densities, and E_f values for the sample groups. The differences in the ARWs and densities between H and L were apparent, but the differences within each grade were small. The densities of H were higher than those of L, and the ARWs of H were narrower than those of L. These results showed that mechanical grading should be useful for sorting lumber according to Young's modulus and for selecting lumber with small, clear wood properties that correlate to wood densities.

The differences in averages and coefficients of variation of ARWs, densities, and E_f values among specimens with varying gauge lengths within a grade were small, as shown in Table 1. The differences in E_f distributions among specimens within a grade were also small, as shown in

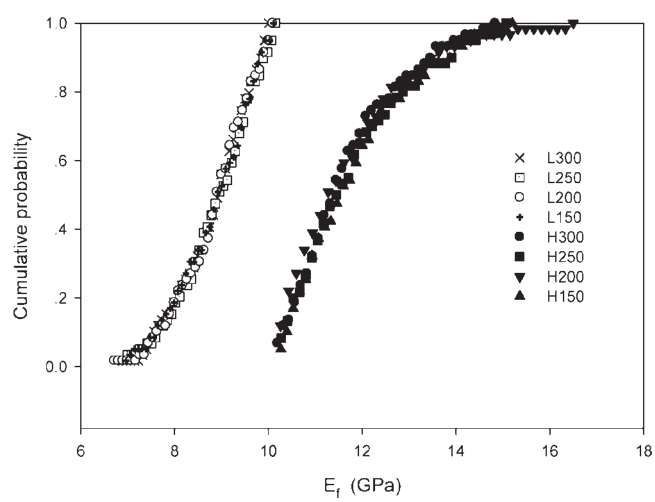

Figure 1.—Distributions of Young's modulus (E_f) by the longitudinal vibration method. Lines are regression curves. Curves on the left are composed of L150, L200, L250, and L300 (low-grade lumber). Curves on the right are composed of H150, H200, H250, and H300 (high-grade lumber).

Table 2.—Basic statistics of tensile strength data.[a]

Specimen	n[b]	Tensile strength		
		Avg (MPa)	SD (MPa)	Skewness
Grade H				
H150	47	32.21	7.56	0.44
H200	52	31.73	8.36	0.97
H250	55	29.12	5.77	0.79
H300	58	27.28	4.90	0.05
Grade L				
L150	41	26.05	5.74	0.18
L200	46	25.51	5.86	−0.35
L250	50	24.65	4.45	−0.46
L300	54	21.41	5.30	0.29

[a] SD = standard deviation; H = high-grade lumber; L = low-grade lumber.
[b] n = number of specimens that failed within the test span.

Figure 1, confirming good matching of the groups. The average E_f values were 11.8 GPa in H and 8.9 GPa in L.

The basic statistics of tensile strength data are presented in Table 2. The distributions of UTS data are illustrated in Figure 2. From these data, it is obvious that increasing the test span lowers the UTS.

Estimating the length effect by the slope method

To estimate the length effect parameters, the NPM (50th percentile) for each sample group was calculated, and each NPM was then plotted in Figure 3a. The length effect parameter s in Equation 1 was then estimated by the least-squares method as shown in Figure 3a. The estimated values of s were 0.188 (1/5.32) in H and 0.226 (1/4.42) in L. Similarly, the NPL (5th percentile) for each sample group was calculated, and the length effect parameter s was estimated as shown in Figure 3b. The s values were 0.185 (1/5.41) in H and 0.318 (1/3.14) in L. For NPM or NPL, the length effect on UTS in H was significantly smaller than that in L. The s value in H was slightly greater for NPM than for NPL.

It was believed that the length effect on UTS should be less in H than in L, because the physical and mechanical properties of H were better than those in L, and that H should have fewer strength-reducing defects, such as knots and other growth defects, than L.

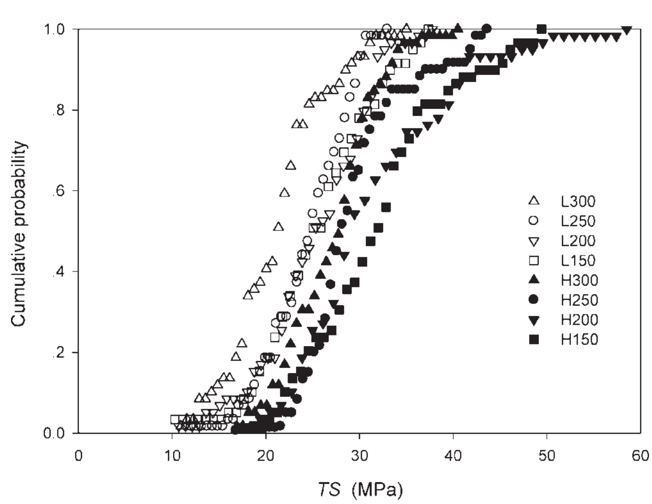

Figure 2.—Distribution of UTS failed within the test span.

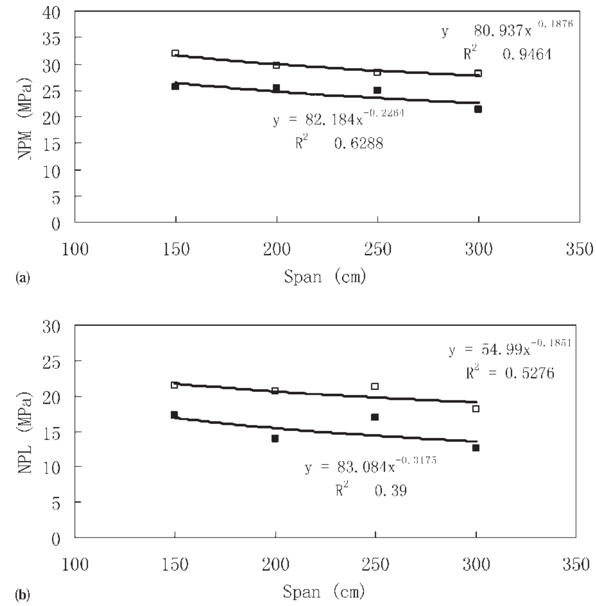

Figure 3.—Relationship between span and UTS. H (open squares) denotes high-grade lumber (H150, H200, H250, and H300). L (filled squares) denotes low-grade lumber (L150, L200, L250, and L300). The 50th percentile (a) and 5th percentile (b) are nonparametric percent point estimates.

Estimating the length effect by the parametric method

When the distribution of UTS may be assumed to be the 2P-Weibull, the shape parameter k of the 2P-Weibull should be the inverse of length effect parameter s, as shown in Equations 1 and 6. There are various fitting methods of distribution function to estimate the parameters of 2P-Weibull, such as the moment method, the regression method, and the maximum likelihood method. Values of the shape parameter k as determined by these three methods were compared, and these values with the inverse of s as obtained by the nonparametric method mentioned above were then compared as well.

The parameter k of the 2P-Weibull fitted by the moment method (2PW-M) can be obtained by

$$\frac{\text{Average}^2}{\text{SD}^2 + \text{Average}^2} = \frac{\Gamma^2\left(1 + \frac{1}{k}\right)}{\Gamma\left(1 + \frac{2}{k}\right)} \tag{9}$$

where $\Gamma(x)$ is the gamma function and Average and SD are the average and standard deviation, respectively, of the UTS distribution as shown in Table 2.

With the k obtained by Equation 9, the parameter m of the 2P-Weibull can be obtained by

$$\text{Average} = m\Gamma\left(1 + \frac{1}{k}\right) \tag{10}$$

The regression method of fitting the 2P-Weibull is applied by first sorting the data, in ascending order, as $x_1, x_2, \ldots, x_n$.

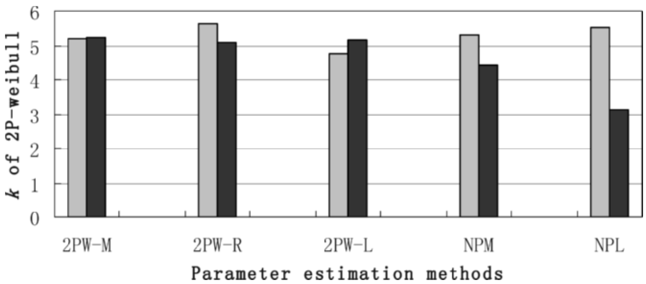

Figure 4.—Shape parameters (k) of 2P-Weibull compared with the inverse of the length effect coefficient s obtained by the nonparametric method. NPM and NPL are the inverse of s for the 50th and 5th percentiles, respectively. For 2PW-M, 2PW-R, and 2PW-L, see Table 3. Lightly shaded bars = H (high-grade lumber) average; heavily shaded bars = L (low-grade lumber) average.

To each of these values a plotting position is assigned: $p_i = i/(n + 1)$. Coordinate pairs (t_i, y_i) are then computed using the transformations $t_i = \ln[-\ln(1 - p_i)]$ and $y_i = \ln x_i$. Once the coordinate pairs (t_i, y_i) have been computed, linear regression is used to estimate the intercept and slope parameters a and b, respectively, of a straight line of the form $y = a + bt$. The parameters k and m of the 2P-Weibull are finally obtained as $k = 1/b$ and $m = \exp(a)$. This is called 2PW-R in the subsequent discussion.

By the maximum likelihood method, the log-likelihood function for the 2P-Weibull can be written as

$$\ln L = \sum_{i=1}^{n} \ln f(x_i)$$
$$= n \ln k - nk \ln m + (k-1) \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \left(\frac{x_i}{m}\right)^k \quad (11)$$

where $f(x)$ is the probability density function of the 2P-Weibull written as

$$f(x) = \frac{k}{m^k} x^{k-1} \exp\left[-\left(\frac{x}{m}\right)^k\right] \quad (12)$$

Stationary values of k and m in Equation 11 can then be determined.

The parameters k and m estimated by the abovementioned three methods are shown in Table 3. Although small differences were found among the m values estimated by the three methods, the k values in H estimated by the likelihood method were slightly lower than the corresponding k values estimated by the other two methods. The harmonic averages of k for each grade were calculated and then compared with the inverse of the length effect parameter s estimated by the nonparametric method. This comparison is shown in Figure 4. The shape parameter k was almost equal to the inverse of s except for the inverse of s for NPL in L.

Summary and Conclusions

An experimental study was conducted to evaluate the length effect on the parallel-to-grain tensile strength of Chinese fir. The tensile tests were conducted for each of four lengths (gauge lengths: 150, 200, 250, and 300 cm) and for two grades (H and L). The length effect on tensile strength in H was smaller than that in L. For L, the length effect of NPL (5th percentile) was larger than that of NPM (50th percentile), while for H, the length effect of NPL (5th percentile) was almost equal to that of NPM (50th percentile). The size effect factors—defined as the ratio of the strengths when the length has been doubled—were 0.88 in H and 0.80 in L for NPL (5th percentile). These length effect factors for H and L should be used in practical designs.

The inverse of s (length effect parameter) for each NPL and NPM was almost equal to the shape parameter estimated by the parametric method except for the case of NPL in L. We observed that there were larger strength-reducing defects, such as knots, in the specimens that fell into the lower tail of the strength distribution of the L samples compared with the H samples. A model with a better fit in regression would improve the estimate of length effect parameters.

Acknowledgments

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Literature Cited

American Society for Testing and Materials (ASTM). 2000. Standard practice for establishing allowable properties for visually-graded dimension lumber from in-grade tests of full-size specimens. ASTM D1990-00. ASTM, West Conshohocken, Pennsylvania.

Table 3.—Estimated parameters of 2P-Weibull.[a]

	Moment (2PW-M)		Regression (2PW-R)		Likelihood (2PW-L)	
Specimen	k	m	k	m	k	m
Grade H						
H150	4.67	35.18	5.11	34.92	4.56	35.20
H200	4.29	34.84	4.89	34.41	3.87	34.93
H250	5.86	31.43	6.40	31.17	5.08	31.52
H300	6.65	29.24	6.43	29.24	6.09	29.31
Grade L						
L150	5.21	28.28	5.26	28.22	5.03	28.34
L200	4.98	27.79	4.63	27.88	5.12	27.77
L250	6.48	26.42	6.05	26.52	6.67	26.45
L300	4.60	23.45	4.66	23.34	4.38	23.46

[a] H = high-grade lumber; L = low-grade lumber.

ASTM International. 2003. Standard practice for evaluating allowable properties for grades of structural lumber. ASTM D2915-03. ASTM International, West Conshohocken, Pennsylvania.
Barrett, J. D. 1974. Effect of size on tension perpendicular-to-grain strength of Douglas-fir. *Wood Fiber* 6(2):126–143.
Bohannan, B. 1966. Effect of size on bending strength of wood members. FPL RP-56. USDA Forest Service, Forest Products Laboratory, Madison, Wisconsin.
Lam, F. and E. Varoglu. 1990. Effect of length on the tensile strength of lumber. *Forest Prod. J*. 40(5):37–42.
Madsen, B. 1990. Length effects in 38mm spruce-pine-fir dimension lumber. *Can. J. Civil Eng*. 17(2):226–237.
Madsen, B. 1992. Structural Behavior of Timber. Timber Engineering Ltd., North Vancouver, British Columiba, Canada. pp 237–286.
Madsen, B. and A. H. Buchanan. 1986. Size effects in timber explained by a modified weakest link theory. *Can. J. Civil Eng*. 13(2):218–232.
Taylor, S. E, D. A. Bender, D. E. Kline, and K. S. Kline. 1992. Comparing length effect models for lumber tensile strength. *Forest Prod. J*. 42(2):23–30.