The Theory and Mathematical Model Underlying the Radial Sawing Simulator—RadSawSim

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Abstract

Gaining additional knowledge of sawing methods that produce sawn boards with a radial grain is a significant problem today because such methods are rarely used. The current technology in Croatian sawmills is adapted to the standard live and cant sawing methods. Experimental in-mill sawing studies designed to monitor yield are subject to financial and technological restrictions. Therefore, simulated sawing is often required. This research presents the calculation method used to formulate a mathematical model that simulates sawing. The objective was to produce as many radial sawn boards as possible.

The need for wood products sorted according to grain orientation (radial sawn boards, half radial sawn boards, flat sawn boards) depends on a number of factors including consumers' preference, current trends, type of final product, where the final product will be assembled, and operational technology, among other things. When there is demand for certain amounts of radial sawn boards and flat sawn boards, the challenge is how to satisfy this demand. The need for radial and flat sawn board processing also depends on a variety of factors. The advantages of radial sawn boards in relation to flat sawn boards are

1. they shrink and expand less in width,
2. they are less likely to distort during drying and steaming,
3. there are fewer board surface cracks during drying and use,
4. they have a clearer sawn surface,
5. they wear equally,
6. the radial grain is accentuated,
7. penetration of liquids is more difficult, and
8. narrow sap-wood in the sawn board is limited to the width of the sap-wood in the log.

However, radial sawn boards also have flaws. Flat sawn boards in relation to radial sawn boards have the following advantages:

1. they are less expensive because production time is less and there is less wood residue,
2. the tangent grain is accentuated,
3. round and oval knots are less visible on the surface of flat sawn boards than the lengthwise positioned knots on radial sawn boards, and
4. flat sawn boards don not collapse as easily as radial sawn boards.

In general, theoretical examination of the various sawmill methods, i.e., setting up an optimal sawblade arrangement, has been explored for a long time. It comes down to a mathematical problem that creates formulas, graphs, and sawblade arrangement albums. The basic log and sawing elements that comprise this mathematical problem are diameter, length, log taper, saw kerf width, dimension of the wood product, and saw blade arrangement, i.e., the method of sawing (Steele 1984).

With the use of the computer, sawing research has become more widespread due to the almost endless possibilities in terms of altering log and sawn product parameters, parameters of sawmill machines, and comparing various sawmill methods. Computer-based evaluations also allow us to take into account certain faults on the logs as well as the optimal position of the log before sawing. In the early 1970s, Lewis and Hallock (1973) developed a computer program BOF (Best Opening Face) designed to

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maximize log volume yield but also lumber and log value yield. The BOF system is a computer simulation model of the sawing process for recovering dimension lumber from sound small-diameter logs. Richards et al. (1979) developed a sawing simulation program that allowed comparison of five sawing methods (quadrant, cant, decision, live, and live sawing plus reripping) and provided the ability to adjust several key sawing parameters including initial placement of the log on the carriage, hidden defects and defect clusters, edging method, rerip location, kerf size, and both centered and off-center core defects. Todoroki (1990) developed the computer-based sawing simulator AUTOSAW that provides graphic visualization of three-dimensional log models. The models can assume both irregular and eccentric forms and incorporate branches at any orientation and differing branch morphologies. Within AUTOSAW the virtual logs and their internal branching structures can be viewed at any rotational setting and at any stage of the sawing process starting from the unsawn log through to individual boards with knot, pith, and wane defects on the four sawn surfaces. AUTOSAW can provide optimal sawing solutions that maximize total lumber volume or value yields. Occena and Schmoldt (1996) developed GRASP, a computer-based interactive graphic sawing program for wood processing, which integrates graphic rendering, solid modeling, and data representation. This program is unique in its flexibility to model just about any sawing operation, from bucking, topping, log breakdown quartering, and veneering to edging, trimming, secondary processing, and even extracting and representing furniture components. Occena et al. (2000) also developed the computer program LogCast, which is a hardwood sawyer training program. The sawing trainer program is a training tool on hardwood log break down for beginning sawyers. The trainee can explore the effects of different sawing patterns, log orientations, and sawing rules of thumb on value and volume recovery. Other sawing simulation computer programs that have been developed include TOPSAW, SAWSIM, OPTITEK, WoodCim, Cut-Log, etc.

The log yield derived in simulated sawing differs somewhat from that obtained in experimental or real sawing in sawmill. The reason is the inability to completely mimic certain real-life conditions in the sawmill when conducting a simulation. The following factors have the most influence on differences between simulated and real sawing in the sawmill: log and sawn product quality (variability within a wood species owing to where the trees that produce logs grow, internal and external wood defects, method of log and wood products grading and measuring), specific characteristics of saw machines and tools (saw kerf width, saw vibration, real world sawing variation, oversizing/undersizing), saw regimes (methods of sawing, capacity), and competency of the machine operator (making a decision for placement of the initial opening sawline or determining the optimum sawblade arrangement).

However, research has concluded that there is a satisfactory level of concurrence in yield between experimental and simulated sawing (Todoroki et al. 2005, Pinto et al. 2006). The difference in yield is diminishing as the new generation of simulation programs more accurately capture log quality characteristics that affect yield such as log curvature, ellipticality, knots, cracks, etc. (Chiorescu and Grönlund 2000).

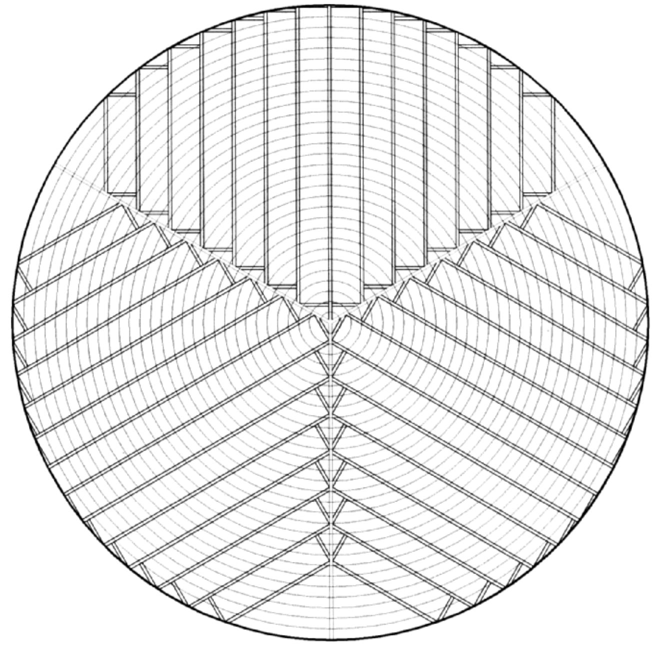

Figure 1.—Illustration of radial log sawing into three sections (thirds).

Motivated by these facts, we thought it would be of value to the industry to develop a mathematical model that simulates sawing using the radial technique, which fits the current demand for radially sawn boards in Croatia and throughout Europe.

The objective of this research was to develop a computer-based mathematical model that simulates sawing logs based on the desire to derive sawn boards with radial grain. The basic requirements of the modeling process included input of log characteristics and sawn board dimensions based on texture (radial sawn board, half radial sawn board, and flat sawn board). The mathematical model calculates the log volume yield, lumber value yield, and log value yield—criteria for choosing the most effective sawing method. The model is able to calculate yields given increases in diameter, log length, log taper, saw kerf width, and thickness of sawn board, with the restriction that all sawn boards are of equal thickness.

Mathematical Model that Simulates Radial Sawing

The principle used to develop the mathematical model is illustrated in the radial log sawing example in Figure 1. These principles can also be used for radial sawing into fourths, fifths, sixths, etc. In other words, in the technological process of sawing logs into radial sawn board, it is assumed that the log would first be sawed on the band saw headrig into sections and then each section would be sawn parallel, along the centerline into sawn boards of equal thickness. Due to technical reasons, when sawing an odd number of sections on the band saw headrig, one section always needs to be sawn through its centerline into two parts, thereby producing an even number of sawn boards.

In order to develop the model, the main sawing zone of each section must be determined. Sawn boards are derived from inside this zone and slabs are generated on the outside. When sawing parallel to the edge, the main zone is

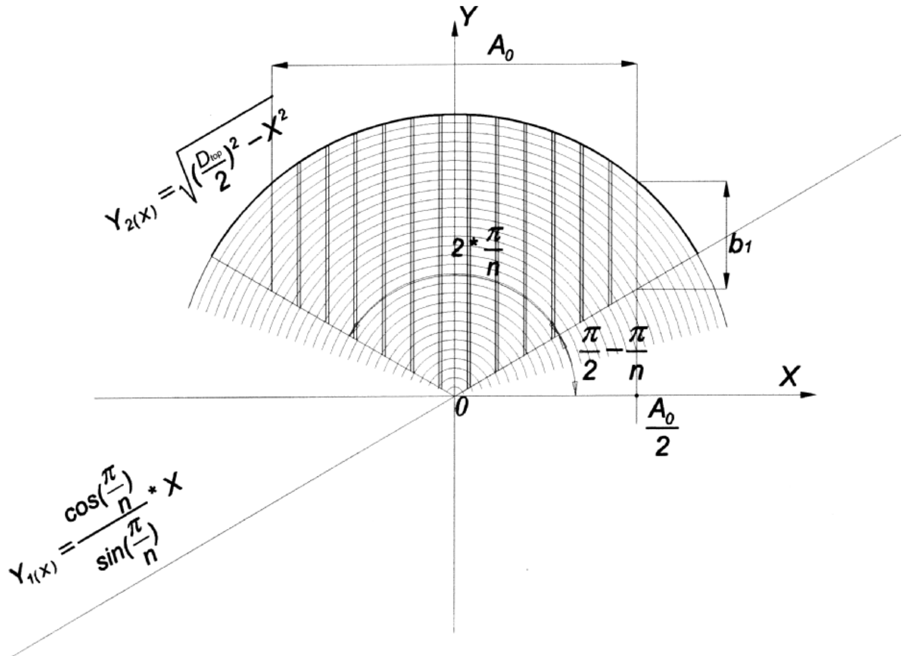

Figure 2.—Log section parameters for calculation of the main zone.

determined by the log's top diameter, the minimal board width with shrinkage allowance, and the number of sections (n) previously sawn from the log. Figure 2 shows the calculation method for Equation 1. The log's main zone section includes the thickness of the sawn board (k_i) with shrinkage allowance and saw kerf width ($k_i - 1$) according to Equation 2.

Sawn boards (k_i) produced with an even number of sections (n) are rounded off to the subsequent lower whole number. Sawn boards (k_i) produced with an odd number of sections (n) are rounded off to the subsequent lower whole number for the first section and the rest ($n - 1$) of the sections (k_i) are rounded off to the subsequent lower whole number. When Equations (1) and (2) are rounded off $A_0 = A_0$ results, i.e., Equation 3.

$$A_0 = \sqrt{D_{\text{top}}^2 - 4 \times b_1^2 \times \sin^2\left(\frac{\pi}{n}\right)} \times \sin\left(\frac{\pi}{n}\right) - 2 \times b_1 \times \sin\left(\frac{\pi}{n}\right) \times \cos\left(\frac{\pi}{n}\right) \quad (1)$$

$$A_0 = k_i \times d_{\text{gross}} + (k_i - 1) \times s \quad (2)$$

$$k_i = \frac{\sqrt{D_{\text{top}}^2 - 4 \times b_1^2 \times \sin^2\left(\frac{\pi}{n}\right)} \times \sin\left(\frac{\pi}{n}\right)}{d_{\text{gross}} + s} - \frac{2 \times b_1 \times \sin\left(\frac{\pi}{n}\right) \times \cos\left(\frac{\pi}{n}\right)}{d_{\text{gross}} + s} + \frac{s}{d_{\text{gross}} + s} \quad (3)$$

where

A_0 = main zone (distance between two symmetrical sawn boards in the sawblade arrangement, which has the minimum width required) (mm);

D_{top} = diameter of the log at the top of its length without bark (cm);

b_1 ($b_{\min}$) = minimal board width with shrinkage allowance, where $b_1 = b_{\min}/2 \rightarrow n = 2$ when live sawing and $b_1 = b_{\min} \rightarrow n = \{3, 4, 5, 6...\}$ when sawing into thirds, fourths, fifths, sixths . . . (mm);

k_i = number of sawn boards in log section;

d_{gross} = thickness of the sawn board with shrinkage allowance (mm); and

s = saw kerf width (mm).

Figure 3 and Equations 4 and 5 show the method for calculating the bearing point coordinates of an individual sawn board at sawing sections of the log for odd and even numbers of sawn boards.

$$X_1 = \frac{1}{2} \times d_{\text{gross}} \times (2j - 3) + (j - 1) \times s$$

$$X_2 = \frac{1}{2} \times d_{\text{gross}} \times (2j - 1) + (j - 1) \times s$$

$$Y_1 = \frac{\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}} \times X_2, \qquad Y_2 = \sqrt{\left(\frac{D_{\text{top}}}{2}\right)^2 - X_2^2}$$

$$j = \left\{1, 2 \ldots \frac{k_i + 1}{2}\right\} \quad (4)$$

$$X_1 = (j - 1) \times d_{\text{gross}} + \left(j - \frac{1}{2}\right) \times s$$

$$X_2 = j \times d_{\text{gross}} + \left(j - \frac{1}{2}\right) \times s$$

$$Y_1 = \frac{\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}} \times X_2, \qquad Y_2 = \sqrt{\left(\frac{D_{\text{top}}}{2}\right)^2 - X_2^2}$$

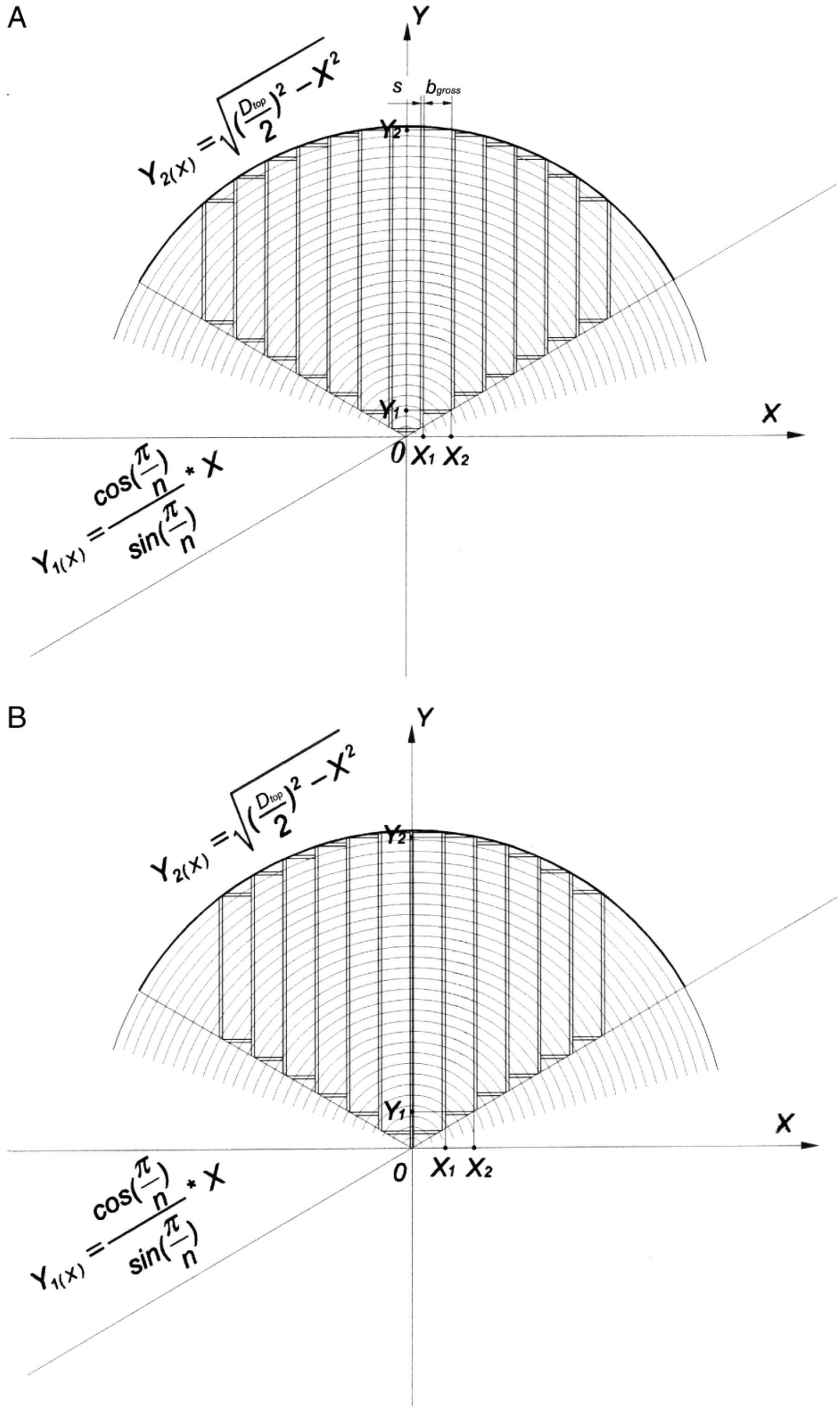

Figure 3.—Bearing point coordinates for individual sawn board at sawing sections of the log in (A) odd number of sawn boards and (B) even number of sawn boards.

$$j = \left\{1, 2 \ldots \frac{k_i}{2}\right\} \tag{5}$$

where

X_1, X_2, Y_1, and Y_2 = coordinate points for individual sawn board,

j = individual sawn board in log section,

D_{top} = diameter of the log at the top of its length without bark (cm),

d_{gross} = thickness of the sawn board including shrinkage allowance (mm), and

s = saw kerf width (mm).

From Equations 1 to 5 it is clear that the log top diameter must be known. In practice, the diameter at the middle of the log's length and the log taper is known (as is the practice in European Union). Therefore, the log's top diameter must be calculated based on the mid-log diameter and log taper. The log taper is expressed as the change in diameter per unit length according to Equation 6. If the geometry of the log with a shortened cone is approximated, then the log top diameter can be calculated using Equation 7.

$$T = \frac{D_{\text{butt}} - D_{\text{top}}}{L} \tag{6}$$

$$D_{\text{top}} = D_{\text{mid}} - \frac{L}{2} \times T \tag{7}$$

where

$T =$ log taper (cm/m),
$D_{\text{top}} =$ diameter of the log at the top of its length without bark (cm),
$D_{\text{mid}} =$ diameter of the log middle of its length without bark (cm), and
$L =$ log length (m).

Log yield volume is defined as the proportion of total sawn board volume relative to the total log volume for a given lumber moisture content (Eq. 8). Total sawn board volume of nominal dimensions at a given state of moisture is calculated according to Equation 9. In order to calculate the log's volume yield, the log's volume must be known. It can be seen that the log's volume converges to the generally known Equation 10 that calculates the log volume when the log taper comes closer to zero.

$$Y_{\text{volume}} = \frac{V_{\text{board}}}{V_{\text{log}}} \tag{8}$$

$$V_{\text{board}} = V_{\text{RB}} + V_{\text{HRB}} + V_{\text{FB}} \tag{9}$$

if Taper = 0

$$V_{\text{log}} = \frac{D_{\text{mid}}^2 \times \pi}{4} \times L \tag{10}$$

where

$Y_{\text{volume}} =$ log volume yield (m^3 sawn boards/m^3 log),
$V_{\text{board}} =$ total sawn board volume of nominal (net) dimensions at a given state of moisture (m^3),
$V_{\text{RB}} =$ radial sawn board volume (m^3),
$V_{\text{HRB}} =$ half radial sawn board volume (m^3),
$V_{\text{FB}} =$ flat sawn board volume (m^3),
$V_{\text{log}} =$ log volume (m^3),
$D_{\text{mid}} =$ diameter of the log middle of its length without bark (cm),
$L =$ log length (m), and
$T =$ log taper (cm/m).

Three different mathematical models for sawn board volume are use depending on the number of sawn log sections (n) and the number of sawn boards in the log sections (k_i):

1. for k_i an even number of boards and n can be an even or odd number of sections,
2. for k_i an odd number of boards and n an even number of sections, and
3. for k_i an odd number of boards and n an odd number of sections.

To determine the sawn board volume, the nominal dimensions of the boards must be known, i.e., dimensions in dry condition. In practice, due to practicality and simplicity, the coefficient of tangential shrinkage must be taken into consideration when determining the net sawn board thickness, regardless of the sawn board's grain according to Equations 11 and 12. In calculating the sawn board volume, the net width of the sawn board is taken into account, which is dependent on annual ring orientation on the crosscut of the sawn board according to Equations 13 to 15. Therefore, the average annual ring orientation on the cross cut of the sawn board must be known, which depends on its position inside the log and the shrinkage coefficient for the given average annual ring orientation inside the log.

$$d_{\text{gross}} = d_{\text{net}} \times \left(\frac{100}{100 - \frac{\beta_t}{\text{GSP}} \times (u_1 - u_2)} \right) \tag{11}$$

$$d_{\text{net}} = d_{\text{gross}} \times \left(1 - \frac{\frac{\beta_t}{\text{GSP}} \times (u_1 - u_2)}{100} \right) \tag{12}$$

$$b_{\text{gross}\,j}^{\text{even-even-odd}} = \sqrt{\left(\frac{D_{\text{top}}}{2}\right)^2 - \left(s \times \left(j - \frac{1}{2}\right) + j \times d_{\text{gross}}\right)^2} - \frac{\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}} \times \left(s \times \left(j - \frac{1}{2}\right) + j \times d_{\text{gross}}\right) \tag{13}$$

$$b_{\text{gross}\,j}^{\text{odd-even}} = \sqrt{\left(\frac{D_{\text{top}}}{2}\right)^2 - \left(\frac{1}{2} d_{\text{gross}} \times (2j-1) + (j-1) \times s\right)^2} - \frac{\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}} \times \left(\frac{1}{2} d_{\text{gross}} \times (2j-1) + (j-1) \times s\right) \tag{14}$$

$$b_{\text{net}} = b_{\text{gross}} \times \left(1 - \frac{\frac{\beta_{\bar{\phi}}}{\text{GSP}} \times (u_1 - u_2)}{100} \right) \tag{15}$$

where

$d_{\text{gross}} =$ (gross) sawn board thickness with shrinkage allowance (mm),
$d_{\text{net}} =$ nominal (net) sawn board thickness (mm),
$b_{\text{net}} =$ nominal (net) sawn board width (mm),
$b_{\text{gross}\,j}^{\text{even-even-odd}} =$ (gross) sawn board width with shrinkage allowance (even number sawn boards when even or odd log sections are present) (mm),

$b_{\mathrm{gross}\,j}^{\text{odd-even}}$ = (gross) sawn board width with shrinkage allowance (odd number sawn boards when even log sections are present) (mm),

b_{gross} = (gross) sawn board width with shrinkage allowance (mm),

u_1 = moisture content in fresh felled wood (%),

u_2 = moisture in wood after drying (%),

GSP = grain (fiber) saturation point,

$\beta_{\bar{\theta}}$ = shrinkage coefficient in a given average annual ring orientation inside the sawn board,

β_t = coefficient of tangential shrinkage,

D_{top} = diameter of the log at the top of its length without bark (cm), and

s = saw kerf width (mm).

It is assumed that $\beta_r = \beta_{\min}$ and $\beta_t = \beta_{\max}$. The shrinkage coefficients, β_r and β_t, arise in mutually perpendicular planes. From these two conditions, the assumption that elliptic changes from the radial to tangential shrinkage coefficient is enforced according to Figure 4.

$$\beta_{\bar{\phi}} = \frac{\beta_r \times \beta_t}{\sqrt{\beta_r^2 \times \sin^2\bar{\phi} + \beta_t^2 \times \cos^{\bar{\phi}}}}, \qquad \forall\bar{\phi} = \left[0, \frac{\pi}{2}\right] \quad (16)$$

where

β_t ($\beta_{\max}$) = coefficient of tangential shrinkage of wood,

β_r ($\beta_{\min}$) = coefficient of radial shrinkage of wood, and

$\bar{\phi}$ = average angle closed by the normal on the annual ring and wider side of the sawn board (radian).

From Equation 16 it is evident that the average angle ($\bar{\phi}$) is needed which closes the normal on the annual ring in the representative point and the wider side of the sawn board (Fig. 5). This angle depends on the position of the sawn board within the log. The average log cross section can be mathematically described using the scalar field ($U_{(X,Y)}$) concentric rings that represent annual rings. The gradient of that scalar field is the vertical vector field on the line level (annual ring border in our case) of the scalar field. The vector of that vector field is found in the normal of the annual ring at a particular point. This vector with any vector parallel to the wider side of the sawn board closes the angle (ϕ).

Given the average angle that closes the normal on the annual ring and the sawn board's wider side, sawn boards are divided into:

- radial sawn board where $\bar{\phi}$ = [0°; 30°], i.e., [0; (π/6)]
- half radial sawn board where $\bar{\phi}$ = [30°; 60°], i.e., [(π/6); (π/3)]
- flat sawn board where $\bar{\phi}$ = [60°; 90°], i.e., [(π/3); (π/2)]

With the average angle ($\bar{\phi}$) that is closed by the normal on the representative annual ring and wider side of the sawn board, we receive the integrational angle (ϕ) along the entire surface of the sawn board's cross section observed according to Figure 6 and Equation 17.

$$\bar{\phi} = \frac{\int_{Y_1}^{Y_2}\left(\int_{x_1}^{x_2} \arccos\left(\frac{Y}{\sqrt{X^2+Y^2}}\right) \times d_x\right) \times d_y}{(X_2 - X_1) \times (Y_2 - Y_1)} \quad (17)$$

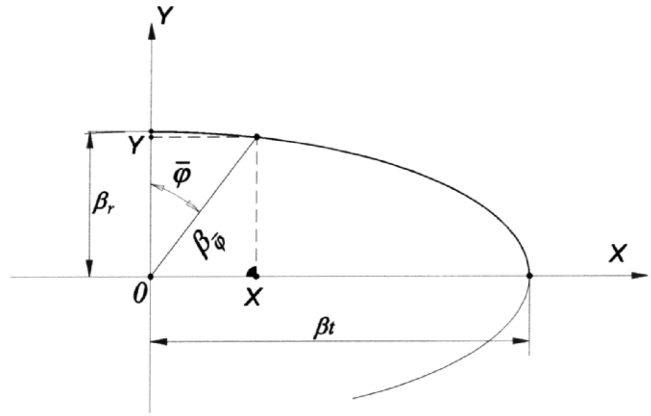

Figure 4.—Geometric illustration of elliptic change from radial to tangential shrinkage coefficient.

where

X_1, X_2, Y_1, and Y_2 = coordinate points for individual sawn board.

It is evident from the previous equations that in order to determine volume and especially value yield, it is necessary to separate the volume of radial sawn boards, half radial sawn boards, and flat sawn boards from the total sawn board volume. It was already mentioned that the average angle is the criteria to separate sawn boards which close the normal at the annual ring and the wider side of the sawn board. In this way, the radial, half radial, and flat sawn board volume can be calculated according to Equations 18 to 27.

$$V_{\mathrm{board}\,j} = d_{\mathrm{net}} \times b_{\mathrm{net}\,j} \times L_{\mathrm{board}} \quad (18)$$

$$V_{\mathrm{RB}}^{\text{even-even-odd}} = 2 \times n \times \sum_{j=1}^{j_{\mathrm{RB}}} V_{\mathrm{board}\,j}^{\text{even-even-odd}} \quad (19)$$

$$V_{\mathrm{HRB}}^{\text{even-even-odd}} = 2 \times n \times \sum_{j=j_{\mathrm{RB}}+1}^{j_{\mathrm{HRB}}} V_{\mathrm{board}\,j}^{\text{even-even-odd}} \quad (20)$$

$$V_{\mathrm{FB}}^{\text{even-even-odd}} = 2 \times n \times \sum_{j=j_{\mathrm{HRB}}+1}^{k_i/2} V_{\mathrm{board}\,j}^{\text{even-even-odd}} \quad (21)$$

$$V_{\mathrm{RB}}^{\text{odd-even}} = n \times \left(\sum_{j=1}^{j_{\mathrm{RB}}} V_{\mathrm{board}\,j}^{\text{odd-even}} + \sum_{j=2}^{j_{\mathrm{RB}}} V_{\mathrm{board}\,j}^{\text{odd-even}}\right) \quad (22)$$

$$V_{\mathrm{HRB}}^{\text{odd-even}} = 2 \times n \times \sum_{j=j_{\mathrm{RB}}+1}^{j_{\mathrm{HRB}}} V_{\mathrm{board}\,j}^{\text{odd-even}} \quad (23)$$

$$V_{\mathrm{FB}}^{\text{odd-even}} = 2 \times n \times \sum_{j=j_{\mathrm{HRB}}+1}^{(k_i+1)/2} V_{\mathrm{board}\,j}^{\text{odd-even}} \quad (24)$$

$$V_{\mathrm{RB}}^{\text{odd-odd}} = 2 \times \sum_{j=1}^{j_{\mathrm{RB}_i}} V_{\mathrm{board}\,j}^{\text{even-even-odd}} + (n-1) \times \left(\sum_{j=1}^{j_{\mathrm{RB}_2}} V_{\mathrm{board}\,j}^{\text{odd-even}} + \sum_{j=2}^{j_{\mathrm{RB}_2}} V_{\mathrm{board}\,j}^{\text{odd-even}}\right) \quad (25)$$

$$V_{\mathrm{HRB}}^{\text{odd-odd}} = 2 \times \sum_{j=j_{\mathrm{RB}_1}+1}^{j_{\mathrm{RB}_1}} V_{\mathrm{board}\,j}^{\text{even-even-odd}} + (n-1) \times 2 \times \sum_{j=j_{\mathrm{RB}_2}+1}^{j_{\mathrm{HRB}_2}} V_{\mathrm{board}\,j}^{\text{odd-even}} \tag{26}$$

$$V_{\mathrm{FB}}^{\text{odd-odd}} = 2 \times \sum_{j=j_{\mathrm{HRB}_1}+1}^{(k_i-1)/2} V_{\mathrm{board}\,j}^{\text{even-even-odd}} + (n-1) \times 2 \times \sum_{j=j_{\mathrm{HRB}_2}+1}^{(k_i+1)/2} V_{\mathrm{board}\,j}^{\text{odd-even}} \tag{27}$$

where

$V_{\mathrm{board}\,j}$ = volume (m^3) of even ($V_{\mathrm{board}\,j}^{\text{even-even-odd}}$) or odd ($V_{\mathrm{board}\,j}^{\text{odd-even}}$) number sawn boards when we have even or odd log sections are present (m^3);

$V_{\mathrm{RB}}^{\text{even-even-odd}}$, $V_{\mathrm{HRB}}^{\text{even-even-odd}}$, $V_{\mathrm{FB}}^{\text{even-even-odd}}$ = volume of even number radial, half radial, and flat sawn boards when even or odd log sections are present (m^3);

$V_{\mathrm{RB}}^{\text{odd-even}}$, $V_{\mathrm{HRB}}^{\text{odd-even}}$, $V_{\mathrm{FB}}^{\text{odd-even}}$ = volume of odd number radial, half radial, and flat sawn boards when even log sections are present (m^3);

$V_{\mathrm{RB}}^{\text{odd}-\text{odd}}$, $V_{\mathrm{HRB}}^{\text{odd}-\text{odd}}$, $V_{\mathrm{FB}}^{\text{odd}-\text{odd}}$ = volume of odd number radial, half radial, and flat sawn boards when odd log sections are present (m^3); and

n = number of log sections.

As already mentioned, the sawn board, in respect to grain of the perpendicular cross section, is divided into radial sawn boards, half radial sawn boards, and flat sawn boards. It is assumed that radial sawn boards are most valuable followed by half radial sawn boards, and flat sawn boards are least valuable. The lumber value yield will be greater when the manufactured sawn board value increases, i.e., a greater share of the most valuable sawn boards. In order to determine the lumber value yield, the individual sawn board value must be known in relation to the total volume of the manufactured sawn board. Knowing only the individual sawn board share value is not enough to determine the lumber value yield. The market price of sawn boards of a certain quality also must be known.

In this model the price relation will be called the quality index of a certain type of sawn board and it will measure the quality of individual pieces of sawn boards. If the value 1 is used as the quality index of radial sawn boards (as the most valuable sawn board), then the quality index of the remaining sawn boards is the ratio of the sawn board's price to the price of radial sawn boards. In this way the average lumber value yield of manufactured sawn boards can be calculated in terms of a value coefficient according to Equations 28 and 29.

$$Y_{\text{value lumber}} = \frac{V_{\mathrm{RB}}}{V_{\mathrm{board}}} \times 1 + \frac{V_{\mathrm{HRB}}}{V_{\mathrm{board}}} \times \frac{C_{\mathrm{HRB}}}{C_{\mathrm{RB}}} + \frac{V_{\mathrm{FB}}}{V_{\mathrm{board}}} \times \frac{C_{\mathrm{FB}}}{C_{\mathrm{RB}}} \tag{28}$$

$$Y_{\text{value lumber}\,€} = C_{\mathrm{RB}} \times Y_{\text{value lumber}} \tag{29}$$

where

$Y_{\text{value lumber}}$ = average lumber value yield,

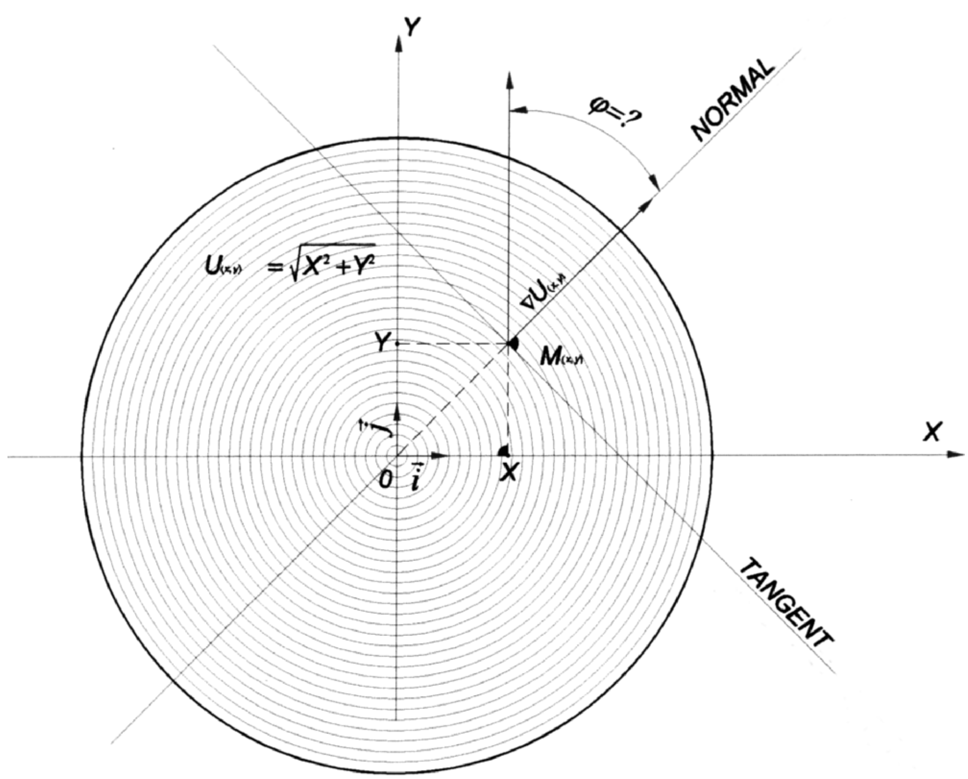

Figure 5.—Geometric illustration of the normal and tangent on the log annual ring shown in the scalar field.

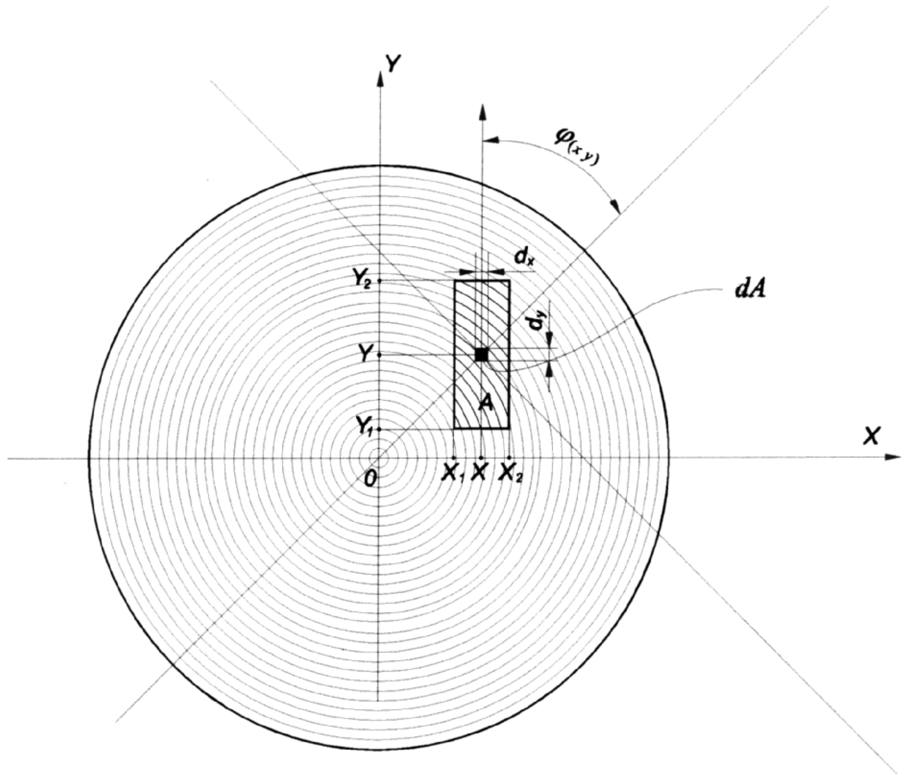

Figure 6.—Geometric illustration of the integrational angle per surface of the sawn board's cross section observed with the objective of determining the average angle.

$Y_{\text{value lumber €}}$ = average monetary value of lumber value yield (€/m^3 of sawn board),

V_{board} = total volume of the manufactured sawn board (m^3),

V_{RB} = radial sawn board volume (m^3),

C_{RB} = price of radial sawn boards (€/m^3 of sawn boards),

V_{HRB} = half radial sawn board volume (m^3),

C_{HRB} = price of half radial sawn boards (€/m^3 of sawn boards),

V_{FB} = flat sawn board volume (m^3), and

C_{FB} = price of flat sawn boards (€/m^3 of sawn boards).

Using only average log and lumber volume yields will not always give a reliable picture concerning the success of a certain method of sawing logs. This is due to the fact that optimal volume and value yields are often obtained using different sawing strategies and thus are out of sync. Therefore, a means of simultaneously considering volume and value yield is needed in modeling log breakdown, and thus the successful sawing method. The indicator that simultaneously takes log volume yield and lumber value yield into consideration is the *coefficient of log value yield*. Log value yield in the form of a coefficient or monetary value can be calculated according to Equations 30 and 31.

$$Y_{\text{value log}} = Y_{\text{volume}} \times Y_{\text{value lumber}} \quad (30)$$

$$Y_{\text{value log €}} = C_{\text{RB}} \times Y_{\text{value log}} \quad (31)$$

where

$Y_{\text{value log}}$ = average log value yield,

$Y_{\text{value log €}}$ = monetary expression of the average log value yield (€/m^3 of log),

Y_{volume} = log volume yield,

$Y_{\text{value lumber}}$ = average lumber value yield, and

C_{RB} = price of radial sawn boards (€/m^3 of sawn boards).

In the previous equations, the price of radial, half radial, and flat sawn boards is determined by the market value and is therefore dependent neither on stereometric characteristics of the log nor the sawing technique. Lumber and log value yield are directly dependent on the sawing technique and are therefore appropriate measures of sawmilling efficiency.

Conclusions and Future Research

The development of the mathematical model, RadSawSim, that simulates sawing using the radial technique enables sawing simulation given varying log dimensions (diameter, log taper, and length), sawing regimes, sawmill machinery, and tools (saw kerf width), as well as sawn board parameters (thickness, wood moisture, minimal width) represents a unique tool for simulating log breakdown. The successful comparison of simulated sawing was enabled through the presentation of volume yield and lumber and log value yield for simulated logs. Implementation of the simulation model, based on varying sawn board parameters and for a number of sawing methods using the radial technique will be presented in a future research paper.

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