Strength Ratios of Knots in Bending for Two Alaskan Softwood Species

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Abstract

The effects of knots on the bending strength of dimension softwood lumber were investigated for two Alaska-grown species, yellow cedar (*Chamaecyparis nootkatensis*) and Sitka spruce (*Picea sitchensis*). An empirical measure of the bending strength ratio for knots was determined by comparison of full-size in-grade test results to corresponding test results from matching small clear specimens. The empirical strength ratios were compared with theoretical strength ratios calculated using existing formulas given in ASTM D245 (ASTM International 2005). It was determined that existing formulas do not provide accurate estimates of bending strength ratio for the two Alaska-grown species under study. Additional predictor variables were used to develop a new model to estimate the influence of knots on the bending strength ratio. Comparison of new formulas to existing formulas demonstrates that this new model is more effective at accounting for variations in bending strength ratio caused by the presence of a knot. It was concluded that the use of the models presented in the 2005 version of the ASTM standards for knot bending strength ratios should be reevaluated. This is especially important for any new timber species being added to the design tables. Species-specific models for estimated bending strength ratios, utilizing additional predictor variables, would help to better predict the effect of knots on bending strength of structural lumber.

The remnants of in-grown branches, commonly known as knots, are one of the most immediately recognizable characteristics of softwood structural lumber. The presence of knots in solid sawn lumber is unavoidable, and knots can be a major factor in reducing the ultimate failure strength of structural softwood lumber. The strength-reducing properties of knots figure heavily into the calculations of lumber design strengths, whether those calculations are based on small clear or in-grade testing procedures (ASTM International 2005). To date, however, little documented experimental work has attempted to accurately quantify strength-reducing effects of knots, especially in species recently added to the design tables. For the present study, we investigated, and sought to accurately quantify, the effect of knots on the bending failure strength of two softwood species grown in Alaska: yellow cedar (*Chamaecyparis nootkatensis*) and Sitka spruce (*Picea sitchensis*).

Predictability of the physical properties of structural materials is one of the cornerstones of structural engineering. Because reliability is so important in this field, the natural characteristics of solid sawn lumber pose unique problems for its use as a structural material. Unique among widely used structural materials, the physical characteristics of dimension lumber are largely formed during the natural growth of the tree and can only be minimally controlled through silvicultural practices. Sawing strategy and various processing techniques during lumber production can be used to minimize, but not to eliminate, the effects of the natural timber characteristics. As a result, lumber is a fundamentally heterogeneous material with physical properties that can vary widely from piece to piece. A large part of structural timber engineering practice has been devoted to developing methods to account for these natural characteristics in the design process. An accurate understanding of the effect of lumber characteristics that act to reduce strength can contribute to the overall efficiency of timber structural design.

Failure testing of full-size structural lumber pieces, commonly referred to as ''in-grade'' testing, is the current

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state of the practice for determining structural lumber design values. Sampling groups and testing procedures for in-grade testing are defined in the ASTM standards for wood construction (ASTM International 2005). As part of the in-grade testing program, strength-reducing characteristics are assigned theoretical ''strength ratios,'' an estimate of the reduction in moment-carrying capacity of the piece that is attributable to the defect. Strength ratios are using in adjusting strength test results as part of the determination of final design strength. Considering the influence of strength ratios on final lumber design strengths, it is important that the assigned strength ratios are accurate. If the actual strength ratios of the defects in the lumber were significantly different from the assigned values, then the adjustments during analysis could produce design values that did not accurately reflect the strength properties of the population in question. For current in-grade analysis procedures, these strength ratios are calculated for knots using formulas given in ASTM D245 (ASTM 2005). However, these formulas can be cumbersome, and they do not account for several observable factors that could be expected to influence structural behavior, including species, knot type, and wood density.

In the present study, an experimental strength ratio was determined by comparing the results of an in-grade test with those of a small clear test on matched specimens collected from the same test piece. Experimental results were compared with the formulas in the 2005 version of ASTM D245. A new model, utilizing additional predictor variables, was developed and compared with the predicted strength ratios as determined by the formulas provided in the 2005 ASTM standard.

Specimen Collection and Preparation

All specimens were collected as part of the Alaskan softwoods in-grade testing program. Lumber was purchased rough green from several different sawmills in southeastern Alaska and shipped to the testing facility in Ketchikan for kiln drying and surfacing to the dimension lumber specifications in place between 2003 and 2005. Samples were collected from random mills in Alaska depending on availability of material. Preliminary grading was conducted at the mill to eliminate specimens that did not meet grade requirements. A trim end was removed from each piece for cutting small clear samples. For tracking purposes, a single specimen number was assigned to both the full-size in-grade specimen and the trim end.

In-grade specimens

Specimens were collected as nominal 2 by 4, 2 by 6, and 2 by 8 boards. Each piece was graded by certified lumber graders in accordance with the rules outlined in Western Lumber Grading Rules 05 (Western Wood Products Association 2004). Width and thickness were measured with handheld calipers (manufactured and calibrated to 0.001 in.) at three points along each piece to determine average width and thickness for each piece. Each piece was assigned a specimen number for tracking.

Small clear specimens

A trim end of each board was used to cut small clear test specimens. The specimens were shaped in accordance with ASTM D143 standards (ASTM 2005) to secondary method size (1 by 1 by 16 in.) for bending tests. Each finished small clear specimen was marked with the specimen number correlating to its full-size piece of origin. The average dimensions of each specimen were measured using digital calipers manufactured and calibrated to 0.001 in.

Testing

In-grade testing

All full-size specimens were tested in bending in accordance with procedures outlined in ASTM D4761 and D198 (ASTM 2005). The temperature of each piece at the time of testing was measured with a Raytek Raynger ST25 infrared temperature meter. Tests were performed on a Metriguard Model 312 Bending Proof Tester in a third-point bending configuration at a span-to-depth ratio of 17. Board lengths extending beyond the outside supports were trimmed to minimize overhang stresses. The ultimate load at failure was used to calculate bending stress according to the elastic behavior stress formula:

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{(P/2)(L/3)}{(bh^2/6)} = \frac{PL}{bh^2} \tag{1}$$

where

$M_{\max}$ = maximum moment in piece,
S = section modulus of piece (in.^{3}),
P = ultimate load at failure (lb),
L = test span length (in.),
b = base (narrow face; in.), and
h = depth (wide face; in.).

Failure coding

After failure, the piece was inspected to determine whether the failure was initiated by a knot. If a knot was judged to be the cause of failure, a failure code was assigned according to codes based on those established in ASTM 4761 (ASTM 2005). The failure code assigned to each piece is intended to reflect the characteristics of the knot judged to be most responsible for initiating failure.

Failures that occurred at nonknot defects were outside the scope of the present study. In-grade specimens that experienced failures initiated at a defect other than a knot, such as at slope of grain or clear wood, were excluded from the study.

Moisture content and specific gravity (full-size specimens)

After failure testing, a clear wood cube was cut from the piece near the failure area and labeled with the specimen identification number. Ovendry specific gravity and moisture content were determined according to Method A as described in ASTM D2395 (ASTM 2005) for each sample.

Small clear testing

All small clear tests were conducted in conformance with ASTM D143 (2005) procedures. Individual temperature readings were not taken for each specimen tested, but the ambient temperature of the laboratory in which the specimens had been stored and tested was monitored. All testing occurred at temperatures of between 63°F and 72°F. Static bending tests were performed on an Instron 300LX hydraulic test frame in the center-point bending configuration. The test jig was provided by Instron and conformed to ASTM D143 specifications. The maximum stress at failure was calculated using a linear elastic stress approach:

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{(P/2)(L/2)}{(bh^2/6)} = \frac{3PL}{2bh^2} \qquad (2)$$

where the variables are as described by Equation 1.

Moisture content (small clear specimens)

After testing, a piece of each small clear specimen was cut, immediately weighed, and labeled with the specimen identification number. These samples were used to determine the ovendry moisture content of each specimen at the time of testing in accordance with the methods outlined in Method B as described in ASTM D4442 (ASTM 2005).

Calculations

Adjustments

According to current procedures for in-grade analysis, it is only necessary to perform a temperature adjustment on the failure stress results if the temperature of the specimen at time of testing is less than 46°F. Because all specimens were tested at temperatures exceeding this cutoff, no temperature adjustments to failure stresses were made. The failure stress of the in-grade test specimens were adjusted to the test moisture content of the matching small clear specimen. Adjustments were calculated according to the formulas outlined in FPL-GTR-126, Section D.6 (US Department of Agriculture [USDA] Forest Products Laboratory 2001)

$$P_2 = \begin{cases} P_1 & \text{for } P_1 \leq 2,415 \text{ psi} \\ P_1 + \left[\dfrac{(P_1 - B_1)}{B_2 - M_1}\right](M_1 - M_2) & \text{for } P_1 > 2,415 \text{ psi} \end{cases} \qquad (3)$$

where

M_1 = moisture content 1,
M_2 = moisture content 1,
P_1 = stress at M_1,
P_2 = stress at M_2,
B_1 = 2,415, and
B_2 = 40.

Strength ratio formulas

Formulas for the strength ratios of knots for full-size lumber pieces tested in bending are given in ASTM D245, Appendix X1 (2005). Knots are classified by three locations as defined in ASTM D245, Section 5.3 (Fig. 1):

- Narrow-face knot: Any knot that shows only on the narrow face of a board, or a knot that shows on both the narrow face and wide face and also contains the intersection of the two faces.
- Wide-face, centerline knot: A knot that shows on the wide face, with the center of the knot farther than two-thirds of the knot diameter from the edge of the piece.
- Wide-face, edge knot: Any knot showing on the wide face of the board, with the center of the knot less than two-thirds of the knot diameter from the edge of the board.

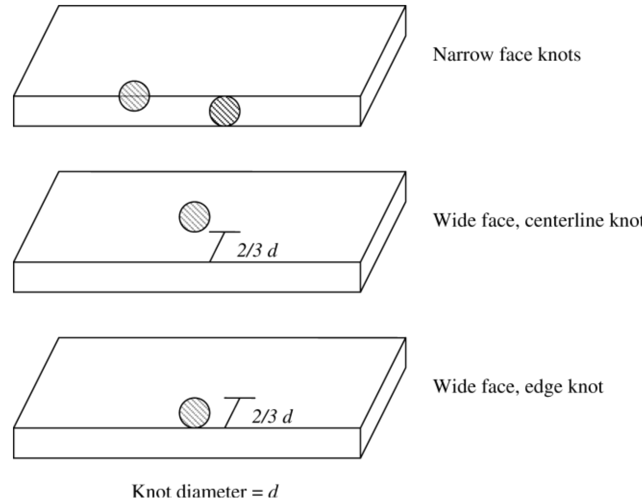

Figure 1.—Knot location classifications for bending specimens.

Separate formulas for strength ratio are given for each knot location. For knots in the narrow face (ASTM D245, Section X1.1, 2005),

$$S = 1 - \frac{k - (1/24)}{b + (3/8)} \qquad S \geq 0.45, \quad b < 6 \text{ in.} \qquad (4)$$

For knots along the centerline of the wide face (ASTM D245, Section X1.2, 2005),

$$S = \begin{cases} 1 - \dfrac{k - (1/24)}{h + (1/2)} & S \geq 0.45,\ 6 \text{ in.} \leq h \leq 12 \text{ in.} \\ 1 - \dfrac{k - (1/24)}{h + (3/8)} & S \geq 0.45,\ h < 6 \text{ in.} \end{cases} \qquad (5)$$

For edge knots on the wide face (ASTM D245, Section X1.3, 2005),

$$S = \begin{cases} \left[1 - \dfrac{k - (1/24)}{h + (1/2)}\right]^2 & S \geq 0.45,\ 6 \text{ in.} \leq h \leq 12 \text{ in.} \\ \left[1 - \dfrac{k - (1/24)}{h + (3/8)}\right]^2 & S \geq 0.45,\ h < 6 \text{ in.} \end{cases} \qquad (6)$$

where, for all three formulas,

S = strength ratio,
k = know size (in.),
b = narrow-face width (in.), and
h = wide-face width (in.).

For strength ratios of less than 0.45, separate formulas are used. No knots in the present study were large enough to result in an estimated strength ratio of less than 0.45, so those formulas were not used in the calculations.

Analysis

All statistical calculations and linear regression models were developed using the R statistical computing environment (R Development Core Team 2007).

Determination of empirical strength ratios

Strength ratios are defined in the ASTM standard as follows: ''Strength ratios associated with knots in bending members have been derived as the ratio of moment-carrying capacity of a member with cross section reduced by the largest knot to the moment-carrying capacity of the member without defect'' (ASTM D245, Section 4.1.1, 2005). This statement is understood to imply that the current strength ratio formulas are based on ratios of experimentally measured moment-carrying capacities. However, published literature discussing the determination of the D245 formulas was scarce. Lacking previously published studies in which to look for precedent, the authors were forced to develop a measurement of experimental moment capacity ratio. In a linear elastic analysis, moment capacity is defined as the moment in a beam at failure and is calculated as

$$M = \sigma(S) \tag{7}$$

where

M = moment (lb·in.),
σ = stress in extreme fiber at centerline (lb/in.^{2}), and
S = section modulus (in.^{3}).

For an accurate determination of knot strength ratios based on moment-carrying capacity, the ideal comparison would be between capacities of two identical test specimens that differ only in the presence or absence of a knot in the cross section. If the wood fiber properties and section moduli were identical, then the difference in moment-carrying capacity between the two could be confidently ascribed to the presence of the knot in the cross section. Unfortunately, this is an entirely theoretical proposition and completely impractical within the context of in-grade testing procedures. A more practical approach is to trim a matching small clear test specimen from each in-grade piece before testing. The failure loads for these small clear samples were used to determine a maximum clear wood fiber stress for each piece based on a linear elastic analysis. It was assumed that the in-grade specimen, if no knot was present in the cross section, would achieve the maximum clear wood fiber stress and a theoretical maximum moment capacity $M^*_{\max}$. This theoretical maximum moment capacity (i.e., the moment capacity that the in-grade piece could be expected to achieve with no knots) can be expressed as

$$M^*_{\max} = \sigma_{\text{small clear}}(S_{\text{in-grade}}) \tag{8}$$

where

$\sigma_{\text{small clear}}$ = stress in extreme fiber at centerline of small clear specimen (lb/in.^{2}), and
$S_{\text{in-grade}}$ = section modulus of in-grade specimen (in.^{3}).

The empirical strength ratio of the knot in the piece was determined by taking the ratio of the measured in-grade moment capacity (with knot) to the theoretical maximum moment capacity (without knot). Again, the ratio is based on elastic principles. So the strength ratio is equivalent to the ratio of the maximum stresses at failure between the in-grade and small clear specimens, expressed as

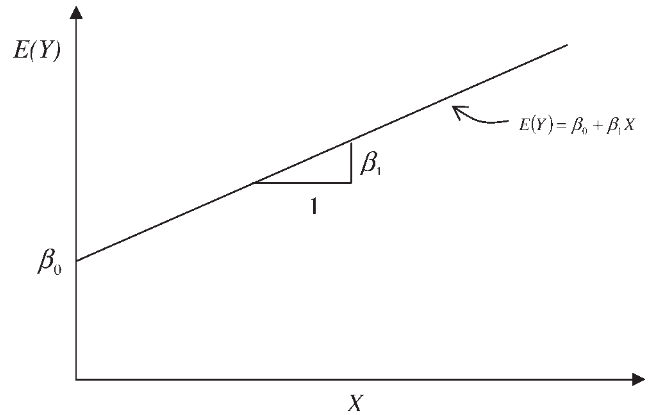

Figure 2.—Coefficients in a linear function.

$$\text{Empirical strength ratio} = \frac{M_{\max}}{M^*_{\max}} = \frac{\sigma_{\text{in-grade}}(S_{\text{in-grade}})}{\sigma_{\text{small clear}}(S_{\text{in-grade}})} = \frac{\sigma_{\text{in-grade}}}{\sigma_{\text{small clear}}} \tag{9}$$

This definition of empirical strength ratio is in agreement with that of a previous study concerning the relationship of flexural strength to strength ratio in southern pine dimension lumber (Doyle and Markwardt 1966).

Linear model interpretation

The determination of strength ratios based on knot size can be viewed as a linear modeling problem. The strength ratio formulas given in ASTM D245 (2005) can be expressed as linear functions of the form

$$E(Y) = \beta_0 + \beta_1 X \tag{10}$$

where $E(Y)$ is the expected strength ratio value and X is knot size in inches. For this type of simple linear function, the coefficients β_0 and β_1 can be interpreted as the intercept and slope of the line defined by Equation 10 in the X-versus-$E(Y)$ plane (Fig. 2). The strength ratio formulas presented in ASTM D245 all share this basic form, although the values of the β_0 and β_1 coefficients change according to narrow-face thickness (for narrow-face knots) or wide-face width (for wide-face knots). The differences in the ASTM D245 formulas for pieces of different dimensions indicate that slightly different linear models are being used in each case. Linear formulations of the equations for knot strength ratios given in ASTM D245 were as follows, where *S, b,* and *k* are as defined above: For narrow-face knots,

$$S = \left[1 + \frac{1}{24b + 9}\right] - \left[\frac{1}{b + (3/8)}\right]k, \quad S \geq 0.45,\ b < 6\ \text{in.} \tag{11}$$

For wide-face centerline knots,

$$S = \begin{cases} \left[1 + \frac{1}{24h + 12}\right] - \left[\frac{1}{h + (1/2)}\right]k \\ \quad S \geq 0.45,\ 6\ \text{in.} \leq h \leq 12\ \text{in.} \\ \left[1 + \frac{1}{24h + 9}\right] - \left[\frac{1}{h + (3/8)}\right]k \\ \quad S \geq 0.45,\ h < 6\ \text{in.} \end{cases} \tag{12}$$

For wide-face edge knots,

$$\sqrt{S} = \begin{cases} \left[1 + \frac{1}{24h+12}\right] - \left[\frac{1}{h+(1/2)}\right]k \\ \quad S \geq 0.45,\ 6\,\text{in.} \leq h \leq 12\,\text{in.} \\ \left[1 + \frac{1}{24h+9}\right] - \left[\frac{1}{h+(3/8)}\right]k \\ \quad S \geq 0.45,\ h < 6\,\text{in.} \end{cases} \qquad (13)$$

A note on the wide-face edge knot model

The formula for strength ratio given for wide-face edge knots in ASTM D245 (2005) implies a transformation in the linear model. Instead of a linear model to estimate mean strength ratio (as for narrow-face and wide-face centerline knots), the linear model for wide-face edge knots estimates the square root of the strength ratio.

Standard mean regression techniques assume that the error terms in the model are normally distributed, with a mean of zero and constant variance. Although error normality cannot be assumed to carry through a model transformation, the present study assumes that the error terms in the square root transformation of the linear model for wide-face edge knots are also normally distributed. Residual and normal probability plots of the data confirm that this is a suitable assumption.

Comparison of Experimental Results to Existing Formulas

To determine whether the formulas described in ASTM D245 (2005) are appropriate for the Alaskan species, a comparison was made between the strength ratios determined through testing and those predicted by the D245 formulas. For this purpose, linear models were developed for the empirical strength ratios, and the model formulas were compared with the linear formulations of the equations presented in D245 (as outlined in Equations 11 through 13). For purposes of comparison, the experimental data were divided for analysis into groups according to knot location and board size to match the groups defined by the D245 formulas. Because the model coefficients (β_0 and β_1) for wide-face knots are dependent on the width of the piece, wide-face knot specimens were grouped according to width. The model coefficients for narrow-face knots are dependent only on board thickness. Therefore, regardless of board width, all narrow-face knot specimens can be analyzed together. Groups and the number of specimens falling within each group are shown in Table 1. Separate linear models for each group were developed using the experimental data from the group. This was done by using the least squares regression method and assuming normally distributed error with constant variance.

Table 1.—Numbers of bending strength specimens.

	2 by 4	2 by 6	2 by 8
Wide-face centerline	54	80	111
Wide-face edge	61	67	84
Narrow face	19	24	65

In linear regression analysis, frequentist statistical theory holds that there exists a ‘‘true’’ linear model for a population. This true model is the theoretical linear model that most accurately describes the behavior of the entire population. When a sample of a population is tested, a linear model describing the sample data is generated as an estimated model for the population. The formula for this estimated model is expressed in the form of Equation 10, with coefficients that are considered to be estimates of the true coefficients for the population. The difference between the estimated and true coefficients depends on the accuracy of the estimated model. The estimated model coefficients and their standard deviations can be used to establish a range of possible values for the true coefficients, commonly expressed as confidence intervals. Confidence intervals were generated for the present study based on the Bonferroni adjustment for comparison of multiple parameters as described by Kutner et al. (2005). Upper and lower limits for each coefficient were calculated from the data to produce intervals that contain (to a 95% probability) the true coefficient for the linear model applicable to the entire population. Coefficients from the D245 models were calculated using Equations 11 through 13. Confidence intervals for the coefficient values determined from the test data were compared with the coefficients as determined from the D245 models, and these results are displayed in Table 2.

A D245 model coefficient that was not contained in the corresponding confidence interval indicates, to statistical significance, that the D245 model did not describe the linear behavior of the tested population. By this criterion, the

Table 2.—Comparison of confidence intervals for linear model coefficients of sampled population to ASTM D245 coefficients.

	95% confidence interval	D245 value
Wide-face centerline		
2 by 4		
β_0	0.8228, 1.1384	1.0108
β_1	−0.4049, −0.1198	−0.2581
2 by 6		
β_0	0.6400, 0.8258	1.0069
β_1	−0.1879, −0.0320	−0.1666
2 by 8		
β_0	0.6752, 0.8186	1.0054
β_1	−0.1613, −0.0665	−0.129
Wide-face edge		
2 by 4		
β_0	0.8964, 1.0027	1.0108
β_1	−0.2880, −0.1183	−0.2583
2 by 6		
β_0	0.8177, 0.9287	1.0069
β_1	−0.2316, −0.1126	−0.1665
2 by 8		
β_0	0.7354, 0.8457	1.0054
β_1	−0.1576, −0.0693	−0.1289
Narrow face		
All		
β_0	0.6126, 0.7576	1.0222
β_1	−0.7318, −0.2334	−0.5321

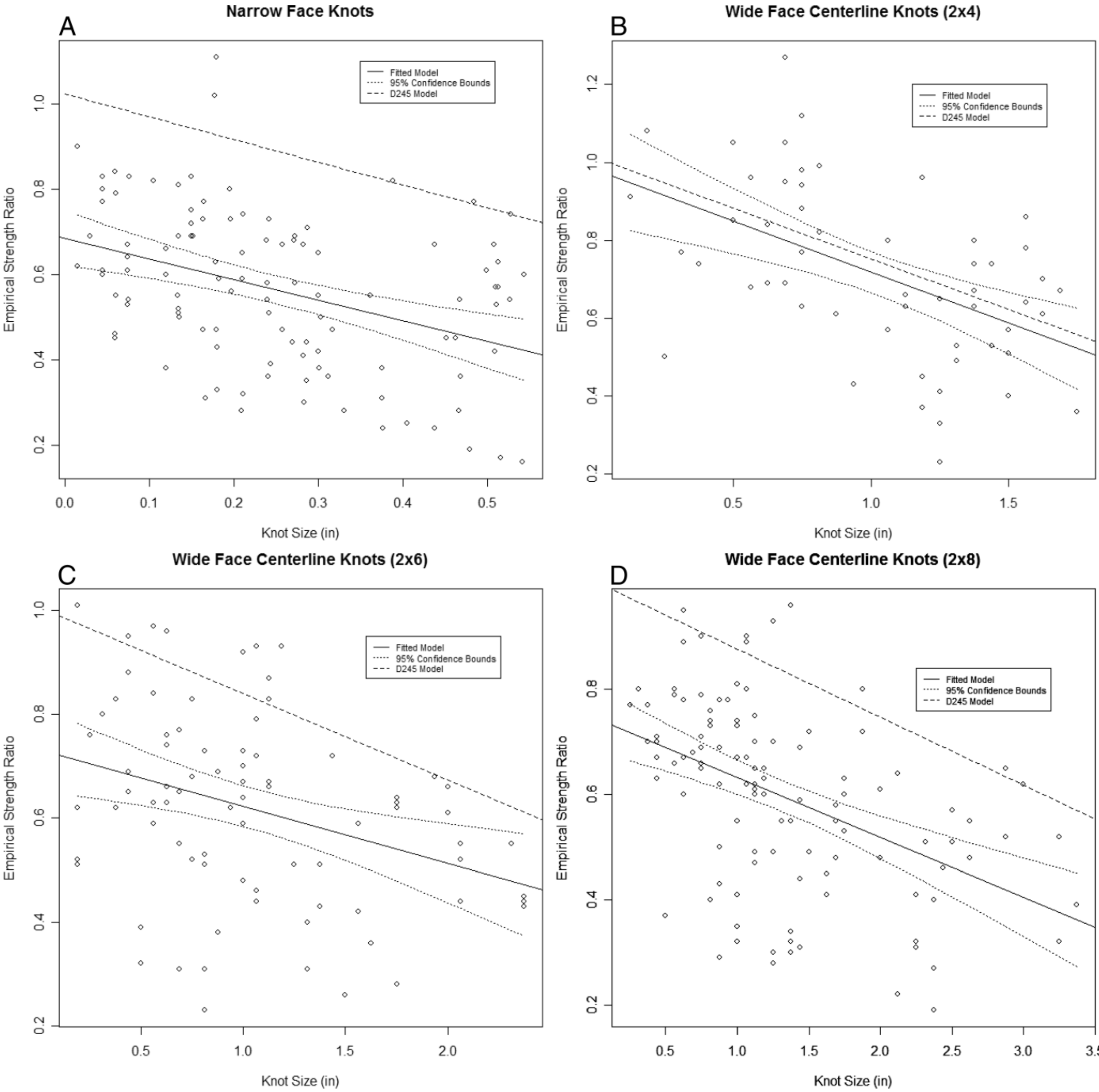

Figure 3.—(A) Plot of empirical bending strength ratios for narrow-face knots. (B) Plot of empirical bending strength ratios for wide-face centerline knots in 2 by 4 specimens. (C) Plot of empirical bending strength ratios for wide-face centerline knots in 2 by 6 specimens. (D) Plot of empirical bending strength ratios for wide-face centerline knots in 2 by 8 specimens.

models defined in D245 were not adequate for six of the seven sample groups. For each failed model, the β_0 coefficient calculated from the D245 formulas was significantly different than the corresponding parameter in the Alaskan species population. However, all D245 β_1 coefficients were contained in their corresponding confidence intervals and could not be considered as significantly different from the parameters defined by the data. This demonstrates a consistent offset in the strength ratio results as determined from the test data from the strength ratios calculated using the D245 formulas. This relationship was clearly seen in the scatter plots of the experimental data, with the fitted regression lines and the lines defined by the linear models outlined in D245 as shown in Figure 3. This demonstrates that the ASTM models as outlined in the 2005 standard are inaccurate in describing the strength ratio value for any given knot size (the β_0 coefficient) and cannot be considered as inaccurate in the description of the relationship between changes in strength ratio and knot size (the β_1 coefficient).

Possible Additional Variables for Predicting Strength Ratio

As discussed above, the formulas as presented in ASTM D245 (2005) could be considered as linear models, with the

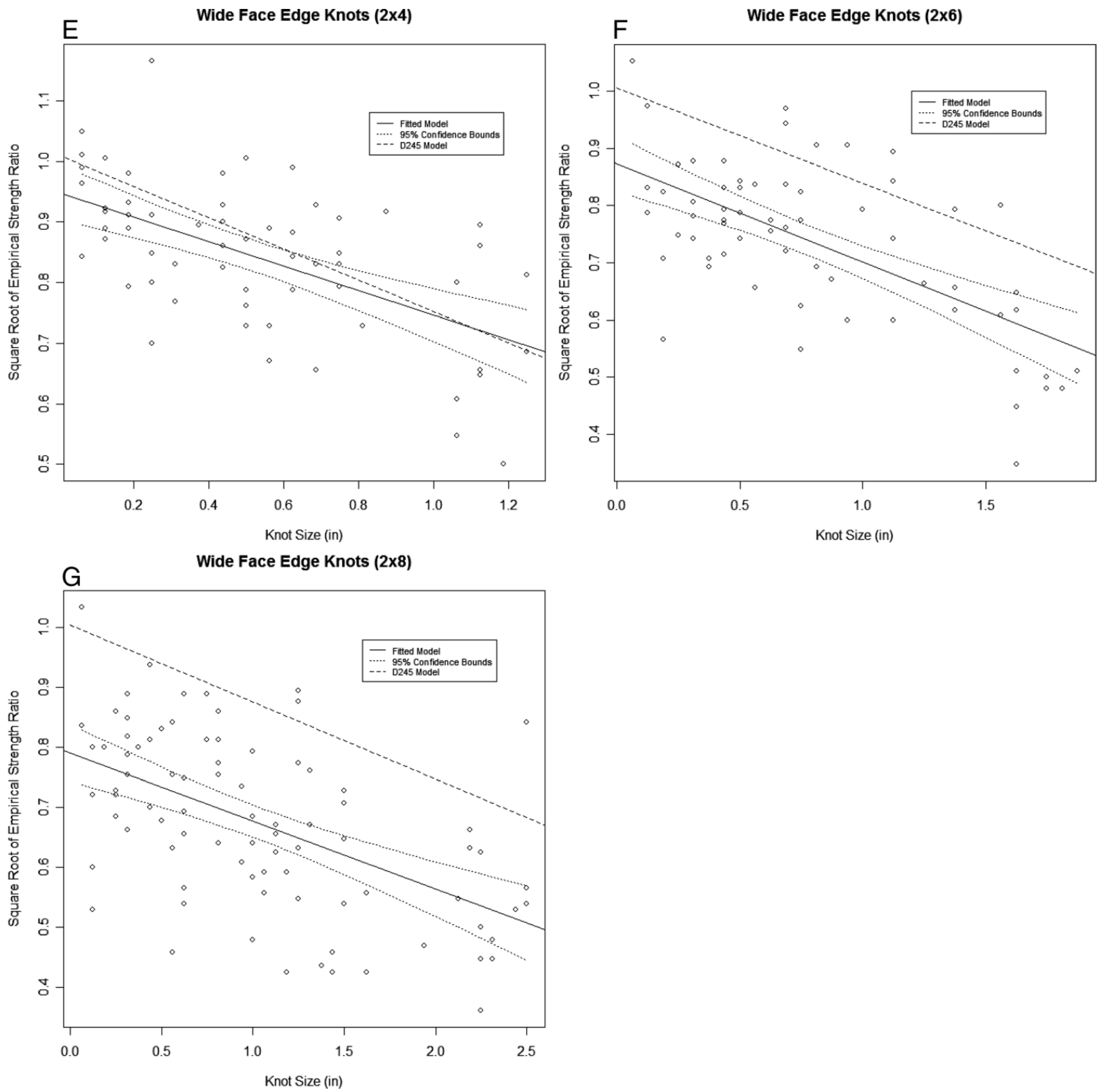

Figure 3, Continued.—(E) Plot of empirical bending strength ratios for wide-face edge knots in 2 by 4 specimens. (F) Plot of empirical bending strength ratios for wide-face edge knots in 2 by 6 specimens. (G) Plot of empirical bending strength ratios for wide-face centerline knots in 2 by 8 specimens.

width and thickness of the piece and the size and location of the knot as predictor variables for the estimated mean strength ratio. In addition to these four predictor variables, several other physical qualities or quantities were measured during in-grade testing that could reasonably be expected to influence the effect of a knot on bending strength:

- Species: The present study includes test results for Alaska-grown yellow cedar and Sitka spruce. Considering differences in cellular structure and branch growth patterns between these species, it is possible that the bending strength reduction caused by the presence of a knot may vary from species to species.
- Grade: The in-grade test results used in the present study include both select structural and No. 2 grade material (Western Wood Products Association 2004). Although knot size is an important criteria for determining lumber grade, other nonknot criteria (e.g., slope of grain and warp), which vary from grade to grade, may have secondary effects on the behavior of knots under load.
- Knot type: The codes used to identify failure knots include information not only on knot location but also on knot type. Knots can be coded as three distinct types:
  - Tight (or intergrown) knots are tied into the surrounding wood grain. There is no separation or discontinuity between the knot and the wood fiber around it.

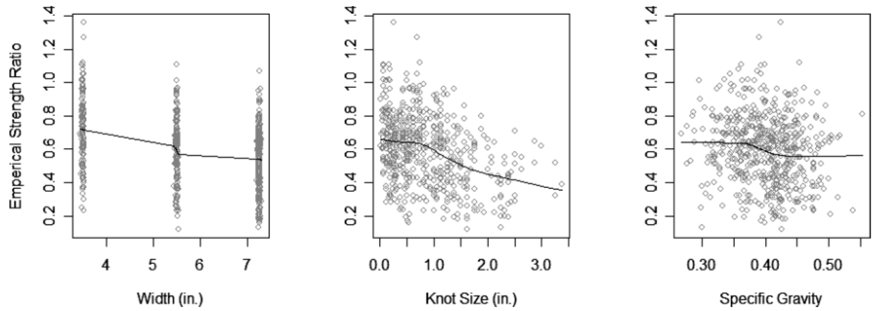

Figure 4.—Lowess fits of continuous predictor variables.

- Loose (or encased) knots are not tied into the surrounding wood fiber. There is a distinct separation of the knot, which causes a break in the wood grain at the knot location. This condition is often the result of a bark-covered branch being grown over and forming a knot. The leftover bark around the branch decays and causes the knot to separate from the wood around it.
- Holes result when a loose knot is knocked out of the piece.

Obviously, the differences in the connections of each knot type to the wood around it would be expected to result in variable behavior under load between the knot types.

- Percentage of knot in cross section: The size of the knot can be measured directly, or the percentage of the piece cross section occupied by the knot can be calculated. This is an alternate measure of the size of the knot, and it may better describe a knot in the piece.
- Specific gravity: The density of the wood fiber, as measure by specific gravity, could be expected to influence how much effect a knot might have on the ultimate bending strength of a piece of lumber.

New Model Development

Because of the consistency of the thickness measurements across the sample, a model based on the present study results cannot describe the effects of changes in thickness. Therefore, thickness was not considered as a predictor variable, and the model developed can be considered as valid only for dimension lumber (nominal 2-in. thickness).

Variable transforms

To search for underlying nonlinear relationships in the data, each continuous variable (width, knot percentage, knot size, and specific gravity) was plotted against empirical strength ratio, and a locally weighted Lowess curve was fit to each plot. A Lowess curve is developed by fitting simple linear models to localized subsets of the data, combining all the local fits, and smoothing the resulting curve. This method can often expose veiled nonlinear relationships in data that are missed by standard regression techniques. A straight Lowess fit indicates that the relationship between the predictor and response variables is fundamentally linear. If the Lowess fit is not linear, it may be possible to transform the data to linearize the relationship between the response and the predictor variable and improve the accuracy of the model. Width, knot size, and specific gravity demonstrated basically linear relationships to empirical strength ratio or had no obvious linearizing transform (Fig. 4). Knot percentage, however, has a distinctly nonlinear relationship to empirical strength ratio that can be improved. A log transform of the knot percentage data relates in a more linear fashion to empirical strength ratio. A Lowess fit for knot percentage and the log transform of the data are shown in Figure 5. When generating a model, the log of the knot percentage was used as a predictor variable.

Initial model selection

Several new models, using all the available predictor variables, were considered for predicting mean bending strength ratio for knots. The goal of model selection was twofold: first, to determine an effective formula for estimated response that accounts for as much of the variation in the experimental results as possible and, second, to develop a capable model that is not unduly complex. As a measure of effectiveness, models were judged by their adjusted R^2 values. The adjusted R^2 value is a measure of how much of the variation in the dependent variable can be explained by the model predictor variables, with an adjustment factor to account for the number of predictor variables used in the model. Several models encompassing all available predictors were initially considered, with continuous terms considered up to the fourth order and both including and excluding interaction effects between the continuous variables. The adjusted R^2 results for all the considered models are shown in Table 3. From these results, it is evident that increasing the complexity of the model by including higher-order terms and interaction effects results in negligible increases in the amount of variability explained by the model. In light of this information, and for simplicity, the decision was made to consider models including only first-order predictor variable terms and no interaction effects. A plot of the residuals of the first-order model

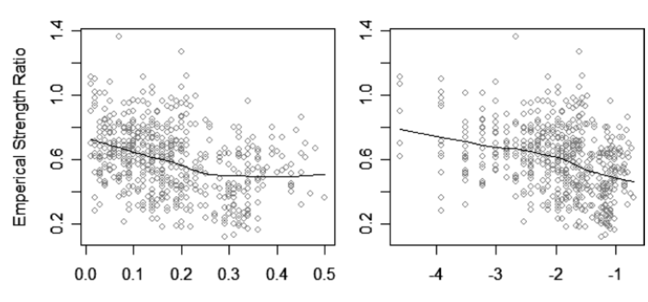

Figure 5.—Lowess fit of original and log-transformed knot percentage.

Table 3.—Comparison of adjusted R^2 values of possible regression models.

Model	Adjusted R^2
Fourth order, with interaction terms	0.4319
Fourth order, no interaction terms	0.4264
Third order, with interaction terms	0.4336
Third order, no interaction terms	0.4281
Second order, with interaction terms	0.4337
Second order, no interaction terms	0.4281
First order, with interaction terms	0.4276
First order, no interaction terms	0.4224

was plotted against the fitted values (Fig. 6). The residuals appear to be distributed randomly around zero, with constant variance, indicating that the assumptions of normality and constant variance were satisfied.

Akaike's information criteria model selection

To further refine the model, the calculation of Akaike's information criteria, as defined in Kutner et al. (2005), was used to differentiate the possible models. Calculations were implemented in R and resulted in the elimination of knot size and specific gravity as predictor variables. All remaining predictor variables were significant to at least the $\alpha = 0.05$ level.

Model comparison

To quantify the amount of variability explained by the ASTM D245 models (2005), the adjusted R^2 value was calculated for each model utilizing the predictor variables from the D245 formulas. These values were compared with the adjusted R^2 value of the new unified model to gauge how much the explanation of variability in knot strength ratio can be improved. A summary of this comparison is provided in Table 4. It is evident that the single new model developed explains more of the variation in empirical knot strength ratio than any of the separate D245 models, in many cases

Figure 6.—Residual plot for first-order model.

Table 4.—Comparison of new model to existing ASTM D245 models.

Model	Adjusted R^2
Unified model	0.4239
Section 1.7.3 models (D245 variables)	
Narrow face	0.1467
Wide face, center, 2 by 4	0.2434
Wide face, center, 2 by 6	0.1062
Wide face, center, 2 by 8	0.2078
Wide face, edge, 2 by 4	0.3283
Wide face, edge, 2 by 6	0.3949
Wide face, edge, 2 by 8	0.2870

by a large margin. This demonstrated that the unified model provided a more accurate estimate of mean knot strength ratio in the Alaskan species while simultaneously reducing the expected variability around the mean.

Final model

The final reduced model can be expressed by the formula:

$$\hat{Y} = 0.751 - 0.084X_1 + 0.105X_2 - 0.047X_3 - 0.106X_4 - 0.052X_5 - 0.007X_6 - 0.051X_7 - 0.087X_8 \quad (14)$$

where

$\hat{Y}$ = expected mean strength ratio in bending;
$X_1 = 1$ if yellow cedar, 0 if Sitka spruce;
$X_2 = 1$ if select structural, 0 if No. 2;
X_3 = width of piece (in.);
$X_4 = 1$ if edge knot, 0 otherwise;
$X_5 = 1$ if narrow-face knot, 0 otherwise;
$X_6 = 1$ if hole, 0 otherwise;
$X_7 = 1$ if intergrown knot, 0 otherwise;
X_8 = log(decimal percentage knot in cross section); and

The final model has an adjusted R^2 value of 0.4239.

Conclusions

An analysis of the experimental results indicated that the formulas given in ASTM D245 for strength ratios of knots in bending do not yield accurate strength ratio values for knots in these species. Although the relationships between the changes in estimated mean strength ratio and the change in knot size were similar to the corresponding parameters in the D245 formulas, a consistent offset indicated that the D245 formulas were overestimating the predicted strength ratio for a given knot size in comparison to the actual value. This inaccurate strength ratio assignment could have an effect on the structural design values for these species, considering the procedure for the analysis of in-grade test results as outlined in FPL-GTR-126.

A new linear model for determining the expected mean strength ratio of a knot was developed that could account for a much larger percentage of the strength ratio variation than the existing models. Whereas the calculation of strength ratios according to the existing D245 formulas required different formulas for different knot locations and lumber sizes, the new model could be expressed as one formula applicable to all knot locations and lumber sizes. In addition, because the new model was developed from tests on Alaska-grown yellow cedar and Sitka spruce, it would produce more accurate predictions of mean knot strength ratios for these species.

Considering the demonstrated inaccuracy of the existing D245 knot strength ratio formulas, the authors have concerns regarding the use of those models to determine strength ratios as part of in-grade lumber design strength calculations. Matched testing and specific strength ratio model development should be made a part of future in-grade testing to ensure that the calculated lumber design strengths reflect the characteristics of the material as accurately as possible.

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